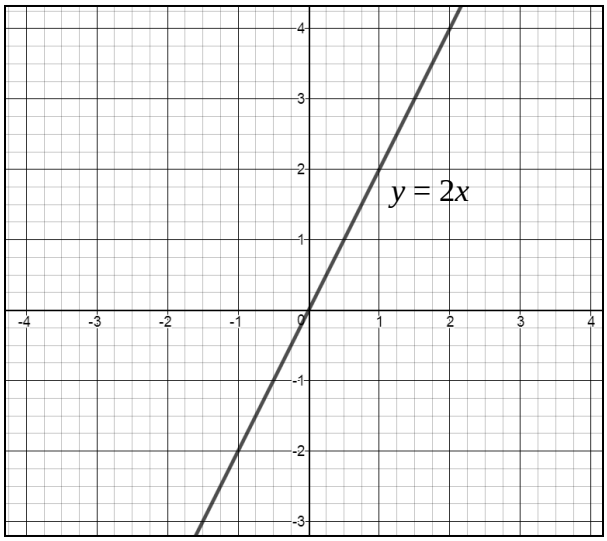
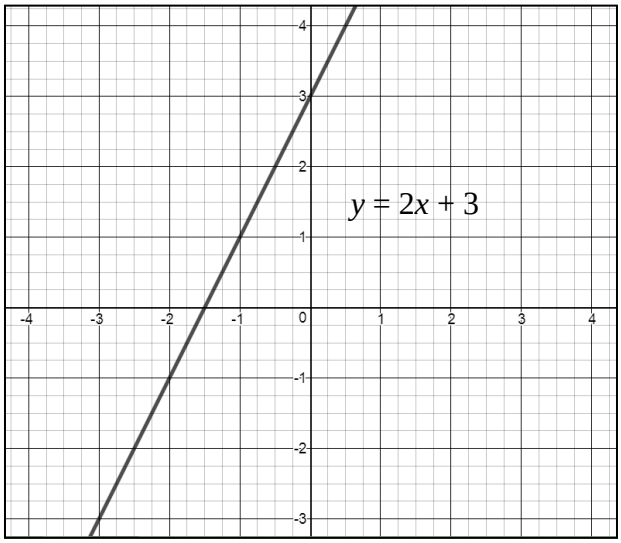
INTRODUCTION TO LINEAR FUNCTIONS

In 7th grade, you spent some time exploring **proportional relationships**. In 8th grade and Algebra it is very important to extend these ideas to investigating **linear relationships**.

Look at the two graphs below. You should recognize the graph on the left as being proportional. The graph on the right is **linear, but not proportional**.





These graphs and equations look similar, but not exactly alike.

Proportional vs Linear: Comparing Graphs

Both graphs are straight lines, but they appear to be in different positions.

As you already know, the proportional graph passes through the origin (0, 0). We are often interested in where a graph crosses the *y*-axis, and a proportional graph does this at $y = 0$. Notice that the linear graph does not cross the y-axis at $y = 0$; it crosses at $y = 3$.

Proportional vs Linear: Comparing Equations

Both equations have a y and $2x$ in them, but the non-proportional graph seems to have an extra term (the "+ 3").

You have learned that the equation of every proportional relationship can be written as $y = kx$. You have called "k" the *constant of proportionality* or *unit rate*. In a linear relationship, we still are interested in what is multiplying the x, but we will be giving it new name.

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Proportional vs Linear: Comparing Tables

This may be the trickiest comparison of the three. Examine the tables below, and the calculations shown to the side.

$y = 2x$

x	y	You have learned that for every pair (except 0,0), $\frac{y}{x}$ always = "k"
0	0	
1	2	$\frac{y}{x} = \frac{2}{1} = 2$
2	4	$\frac{y}{x} = \frac{4}{2} = 2$
3	6	$\frac{y}{x} = \frac{6}{3} = 2$
4	8	$\frac{y}{x} = \frac{8}{4} = 2$

$y = 2x + 3$

x	y	Does $\frac{y}{x}$ always = "k"?
0	3	
1	5	$\frac{y}{x} = \frac{5}{1} = 5$
2	7	$\frac{y}{x} = \frac{7}{2} = 3.5$
3	9	$\frac{y}{x} = \frac{9}{3} = 3$
4	11	$\frac{y}{x} = \frac{11}{4} = 2.25$

$\frac{y}{x}$ keeps equaling a different number, so there **isn't** a constant value of k

You will learn that in linear relationships, $\frac{y}{x}$ does NOT always = k, but there is a ***different*** calculation that is always constant. This calculation is called "rate of change" and is what we'll be exploring for the rest of this investigation.

RATE OF CHANGE

What is it?

A rate of change is a ratio that compares the change in one quantity to the change in another.

A ratio can look like "3 to 5" or 3:5 or $\frac{3}{5}$.
We'll be using fractions.

Watch Video#1 – ***just the first 3 minutes and 20 seconds*** – to see lots of real-life examples of rate of change. https://www.youtube.com/watch?v=yJDsquZTG0A (stop watching at 3:22)

The narrator of the video used LOTS of examples that talked about something changing over time: miles per **hour**, inches per **month**, etc. Keep in mind that rate of change problems aren't required to have time be involved; we could look at miles per gallon or dollars per download

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How to calculate it

Now that you have an idea of what rate of change is, it's time to learn to calculate it.

Video#2 is a great starting place: https://www.youtube.com/watch?v=N4nwM61osHo&t=294s

In Algebra, you will need to be able to find rate of change from a table, graph, equation or even a verbal situation. The remainder of this summer work concentrates on working with tables.

CONTEXT

You may have noticed that in math class, sometimes we work with numbers in a context or situation, and sometimes with just numbers that don't have meaning.

Context	No context
John has 3 apples and Sally has 2 apples. How many apples do they have all together?	What is 3 + 2?

Many examples look at rate of change with NO context, so you should get familiar with what they will look like.

Go back to the Mario/Luigi video you just watched. In the LAST example, the context was a situation involving the height of something (an airplane descending?) changing over time. Look at how we could rephrase the question two different ways – in context and not in context.

Context	No context
A plane was observed to be 600 meters above the ground at 10 minutes. At 50 minutes it was observed to be 400 meters above the ground. What was the rate of change?	Find the rate of change between the points (10, 600) and (50, 400).

RATE OF CHANGE FROM A TABLE

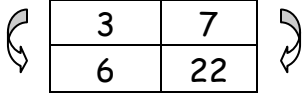
Suppose we want to calculate the rate of change between the points (3, 7) and (6, 22). Finding the rate of change means making a ratio of the change in *y* to the change in *x*. Here's how to do that:

1. Put the points into an *xy* table.

x	*y*
3	7
6	22

2. Find the "change in *x*" - this means find how *x* changed from 3 to 6

   and the "change in *y*" - this means find how *y* changed from 6 to 22

+3 

+15

3. Make the ratio (in fraction form) "change in *y*" to "change in *x*"

$$\frac{ychange}{xchange} = \frac{15}{3} = 5$$

We say the constant rate of change is 5.
For every change in *x* of 1 unit, there is a change in *y* of 5 units.

Watch Video#3 to see another example:
https://www.youtube.com/watch?v=vFXXStHpfaQ&t=47s

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CONSTANT RATE OF CHANGE

A ***constant*** rate of change means that the rate of change, for the particular relationship, is always the same.

It is common that you will be given a table and asked if it shows a "constant rate of change." In this case, you will need to find several rates of change and determine whether they are always the same.

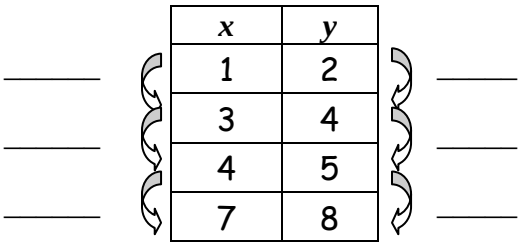
Video#4 will show a few examples: https://www.youtube.com/watch?v=PjiFOzI3Rs4

Notice in that last example that there was NO constant rate of change.

The table below has a new twist. See if you can work through the instructions to determine whether it shows a constant rate of change.

x	*y*
1	2
3	4
4	5
7	8

Step 1: find and write the changes in *x* and the changes in *y* between each row.



Step 2: make 3 fractions by writing each "change in *y*" OVER its partner "change in *x*"

Step 3: compare the fractions – are they all equivalent to the same fraction?

If so, the relationship has constant rate of change and is linear.

If not, the relationship does not have a constant rate of change, and is NOT linear.

☞☞☞ Go to Google Classroom and do Checkpoint #4 ☜☜☜

LINEAR RELATIONSHIP DEFINITION

The definition of a linear relationship is very short, and it requires you to piece together what you have learned so far.

→→A linear relationship is one that has a **constant rate of change**. ←←

We will be comparing how much the **y-value changes** to how much the **x-value changes**.

Video#5 and Video#6 are both excellent recaps of this investigation.

https://www.youtube.com/watch?v=C7CY4l99S7M
https://www.youtube.com/watch?v=T3axRpSZY9M

☞☞☞ Go to Google Classroom and do Checkpoint #5 ☜☜☜

Specific Instructions for INTRODUCTION TO LINEAR FUNCTIONS

Task	Done?
1. Read page 1 Introduction and Proportional vs. Linear: Comparing Graphs and Proportional vs. Linear: Comparing Equations	
2. Do Practice#1. When complete, check with posted answers (link on Google Classroom)	
3. Go to Google Classroom and do Checkpoint #1 (link on Google Classroom)	
4. Read page 2 Proportional vs. Linear: Comparing Tables	
5. Read the very top of page 3 Rate of Change: What is it?	
6. Watch Video#1 (link on Google Classroom)	
7. Do Practice#2. When complete, check with posted answers (link on Google Classroom)	
8. Go to Google Classroom and do Checkpoint #2 (link on Google Classroom)	
9. Read the rest of page 3 Rate of Change: How to calculate it	
10. Watch Video#2 (link on Google Classroom)	
11. Read page 4 Context	
12. Read page 5 Rate of Change from a Table	
13. Watch Video#3 (link on Google Classroom)	
14. Do Practice#3. When complete, check with posted answers (link on Google Classroom)	
15. Go to Google Classroom and do Checkpoint #3 (link on Google Classroom)	
16. Read top of page 6 Constant Rate of Change	
17. Watch Video#4 (link on Google Classroom)	
18. Read remainder of page 6 Constant Rate of Change	
19. Do the sample problem on page 6	
20. Do Practice#4. When complete, check with posted answers (link on Google Classroom)	
21. Go to Google Classroom and do Checkpoint #4 (link on Google Classroom)	
22. Read page 7 Linear Relationship Definition	
23. Watch Video#5 (link on Google Classroom)	
24. Watch Video#6 (link on Google Classroom)	
25. Do Practice#5. When complete, check with posted answers (link on Google Classroom)	
26. Go to Google Classroom and do Checkpoint #5 (link on Google Classroom)	

General Instructions for Algebra Elective Summer Assignment

Welcome to the Algebra Elective class! I am really looking forward to meeting you in September and working with you next year.

To get ready for September, I am asking you to investigate the concept of "rate of change" over this summer. You will be tested on this material on the second day of class. Although the grade on this assessment will not count toward your report card grade, students who do not show mastery (85%) on the assessment will meet with me to discuss ways to improve your performance in the class.

Here are some general guidelines

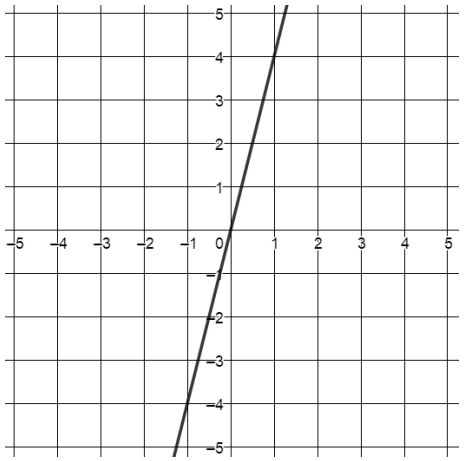
1. Go to Google Classroom and join the class with code 73qlba
2. Read and follow the Specific Instructions carefully (I know the list seems very long; don't panic! Many of the items are actually pretty short).
3. When you are directed to read in the assignment, read carefully and thoughtfully. (That means THINK about what you are reading)
4. When you are directed to watch a video, focus on the video. Do not text your friends or watch TV or listen to music or watch YouTube videos or go on social media or swim in the pool or ….. while you are "watching" the video.
5. If you have any questions during the reading or video, jot them down. Re-read or re-watch to see if you can answer them on your own. Ask me rhekker@nyackschools.org or search for another video to help.
6. PLEASE reach out to me (or ask a parent) if you experience any difficulty with the assignment that you can't address on your own. Do not enter class in September saying "I don't get it." You have two months to ask for help if you need it.

Good luck, and have a great summer!

Practice#1

For each graph shown, tell whether it is Proportional, Linear or Neither. You should be able to explain how you know.

1.



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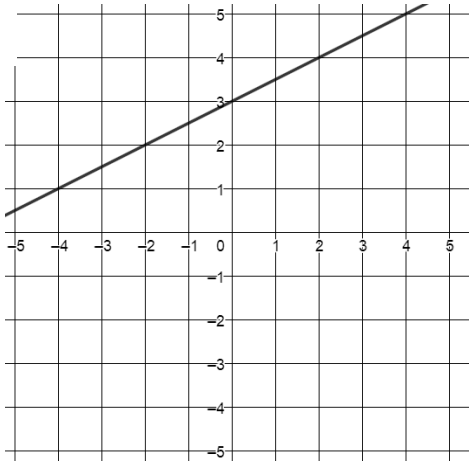

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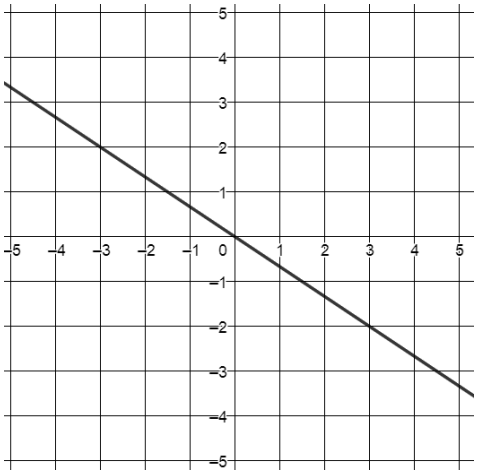

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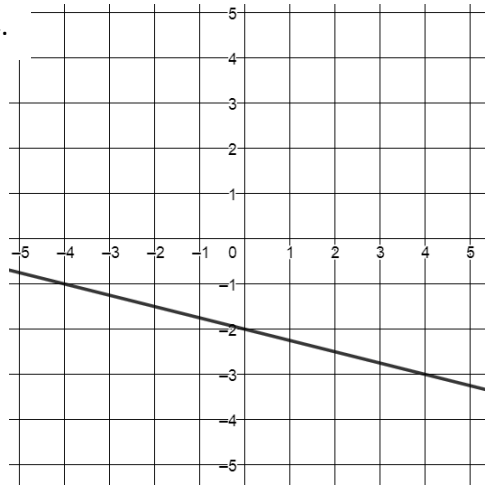

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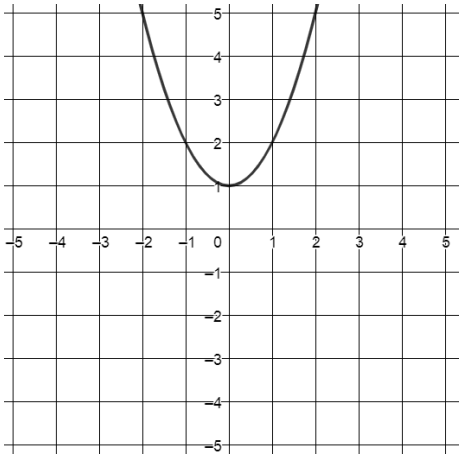

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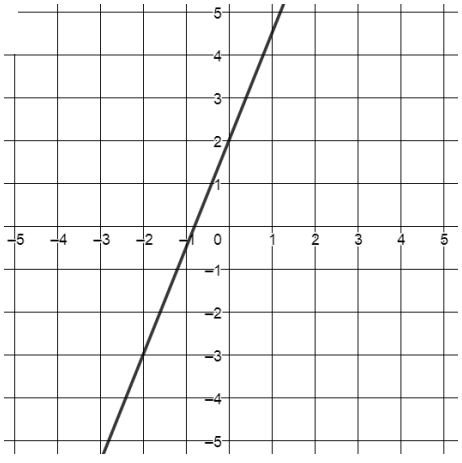


For each equation shown, tell whether it is Proportional, Linear or Neither. You should be able to explain how you know.

7. $y = \frac{3}{5}x$

8. $y = \frac{1}{2}x - 6$

9. $y = x^2 + 3$

10. $y = -3x$

11. $y = x$

12. $y = 2 - x$

Practice#2

Write the rate as it is described in each sentence. **Then** explain what the unit rate means.

Include units!

1. A pencil company used 90 grams of rubber to make 10 pencils.

2. During a lunch rush a fast food restaurant sold 70 sodas and earned $49.

3. A restaurant went through 7 boxes of plastic forks over 90 months.

4. A candy company used 40 gallons of syrup to make 8 batches of candy.

5. John graphed a point on a coordinate grid. To graph a second point, he moved his pencil 2 units up and 5 units to the right. He repeated this process a few more times to graph more points then connected them with a line. The line's rate can be described as "2 units up for every 5 units right."

Practice#3

Find the rate of change for these pairs of points:

2.

x	*y*
2	6
5	21

1.

x	*y*
-2	6
-5	21

4.

x	*y*
6	15
-1	29

3.

x	*y*
-4	-12
0	-8

(If you have trouble getting started on these, put the points into an x-y table)

6. (3, 7) and (10, 7)

5. $\left(\frac{1}{3}, \frac{3}{2}\right)$ and $\left(\frac{7}{3}, \frac{9}{2}\right)$

Practice#4

Decide whether each table is showing a constant rate of change. Show how you decided.

1.

x	*y*
3	25
4	100
5	125
6	150

2.

x	*y*
4	232
6	348
8	464
10	580

3.

x	*y*
3	-9
4	-6
5	-3
6	3

4.

x	*y*
3	-5
4	0
6	5
10	10

5.

x	*y*
0	1
1	2
2	5
3	10

6.

x	*y*
1	2
3	8
6	17
7	20

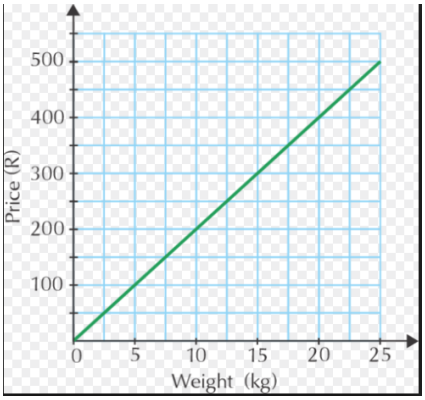
Practice#5

Decide whether each relationship shown below is proportional, linear or neither. Be able to explain how you decided.

1.

x	*y*
0	0
1	1
2	4
3	9

2.



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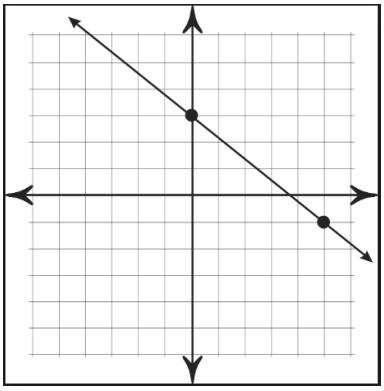

4.

x	*y*
5	-25
3	-15
2	-10
-1	5

5.

x	*y*
-2	7
0	13
2	19
4	25

6.