General Instructions for Algebra Elective Summer Assignment

Welcome to the Algebra Elective class! I am really looking forward to meeting you in September and working with you next year.

To get ready for September, I am asking you to spend time this summer investigating the concept of "rate of change" and connecting it to the idea of a linear relationship. There will be an assessment on this material early in the year. Although the grade on this assessment will not count toward your report card grade, students who do not show mastery (85%) on the assessment will meet with me to discuss ways to improve your performance in the class.

Here are some general guidelines

1. Follow instructions in a separate attachment to log into Schoology for:
    a. an additional copy of this assignment, if you lose this one.
    b. announcements, updates or corrections
    c. information about extra resources
    d. answers to Practice Problems (posted toward the middle of the summer)
2. Read and follow the Specific Instructions (next page) ***carefully*** (I know the list seems very long; don't panic! Many of the items are actually pretty short).
3. When you are directed to read in the assignment, read carefully and thoughtfully. (That means THINK about what you are reading)
4. When you are directed to watch a video, focus on the video. Do not text your friends or watch TV or listen to music or watch YouTube videos or go on social media or swim in the pool or ….. while you are "watching" the video.
5. If you have any questions during the reading or video, jot them down. Re-read or re-watch to see if you can answer them on your own. Ask me at rhekker@nyackschools.org or search for another video to help.
6. PLEASE reach out to me (or ask a parent) if you experience any difficulty with the assignment that you can't address on your own. Do not enter class in September saying "I don't get it." You have two months to ask for help if you need it.

Good luck, and have a great summer!

Specific Instructions for INTRODUCTION TO LINEAR FUNCTIONS

Task	Done?
1. Read the very top of page 3 Rate of Change: What is it?	
2. Watch Video#1 (link in Schoology)	
3. Read the rest of page 3 Rate of Change: How to calculate it	
4. Watch Video#2 (link in Schoology)	
5. Read page 4 Context	
6. Read page 5 Rate of Change from a Table; try the sample problem	
7. Watch Video#3 (link in Schoology)	
8. Do Practice#1. When complete, check with posted answers (link on Schoology)	
9. Go to Schoology and do Checkpoint #1	
10. Click "View Score" – if you got any questions wrong, make sure you understand your mistakes	
11. Read top of page 7 Constant Rate of Change	
12. Watch Video#4 (link in Schoology)	
13. Read remainder of page 7 Constant Rate of Change; try the sample problem	
14. Do Practice#2. When complete, check with posted answers (link on Schoology)	
15. Go to Schoology and do Checkpoint #2	
16. Click "View Score" – if you got any questions wrong, make sure you understand your mistakes	
17. Read page 9 Linear Relationship Definition	
18. Read page 10 Introduction and Proportional vs. Linear: Comparing Graphs and Proportional vs. Linear: Comparing Equations	
19. Do Practice#3. When complete, check with posted answers (link on Schoology)	
20. Go to Schoology and do Checkpoint #3	
21. Click "View Score" – if you got any questions wrong, make sure you understand your mistakes	
22. Read page 13 Proportional vs. Linear: Comparing Tables	
23. Watch Video#5 (link in Schoology)	
24. Watch Video#6 (link in Schoology)	
25. Do Practice#4. When complete, check with posted answers (link on Schoology)	
26. Go to Schoology and do Checkpoint #4	
27. Click "View Score" – if you got any questions wrong, make sure you understand your mistakes	
28. Read pages 15 and 16 Equations of Lines: Putting it all together	
29. Do Practice#5 When complete, check with posted answers (link on Schoology)	
30. Go to Schoology and do Checkpoint #5	

RATE OF CHANGE

What is it?

A rate of change is a ratio that compares the change in one quantity to the change in another.

A ratio can look like "3 to 5" or 3:5 or $\frac{3}{5}$.
We'll be using fractions.

Watch Video#1 – ***just the first 3 minutes and 20 seconds*** – to see lots of real-life examples of rate of change. https://www.youtube.com/watch?v=yJDsquZTG0A (stop watching at 3:22)

The narrator of the video used LOTS of examples that talked about something changing over time: miles per **hour**, inches per **month**, etc. Keep in mind that rate of change problems aren't required to have time be involved; we could look at miles per gallon or dollars per download

RATE OF CHANGE

How to calculate it

Now that you have an idea of what rate of change is, it's time to learn to calculate it.

Video#2 is a great starting place: https://www.youtube.com/watch?v=N4nwM61osHo&t=294s

In Algebra, you will need to be able to find rate of change from a table, graph, equation or even a verbal situation. The remainder of this summer work concentrates on working with tables.

CONTEXT

You may have noticed that in math class, sometimes we work with numbers in a context or situation, and sometimes with just numbers that don't have meaning.

Context	No context
John has 3 apples and Sally has 2 apples. How many apples do they have all together?	What is 3 + 2?

Many examples look at rate of change with NO context, so you should get familiar with what they will look like.

Go back to the Mario/Luigi video you just watched. In the LAST example, the context was a situation involving the height of something (an airplane descending) changing over time. Look at how we could rephrase the question two different ways – in context and not in context.

Context	No context
A plane was observed to be 600 meters above the ground at 10 minutes. At 50 minutes it was observed to be 400 meters above the ground. What was the rate of change?	Find the rate of change between the points (10, 600) and (50, 400).

In the descending airplane context, we are interested in how height is changing compared to (or depending on) how time is changing. We let time be the independent variable (x) and height be the dependent variable (y), and then proceed to calculating rate of change.

How exactly do we do that? Keep reading!

RATE OF CHANGE FROM A TABLE

Suppose we want to calculate the rate of change between the points (3, 7) and (6, 22). Finding the rate of change means making a ratio of the change in *y* to the change in *x*. Here's how to do that:

1. Put the points into an *xy* table.

x	*y*
3	7
6	22

2. Find the "change in *x*" - this means find how *x* changed from 3 to 6

   and the "change in *y*" - this means find how *y* changed from 6 to 22

+3 | *x* | *y* | +15

x	*y*
3	7
6	22

3. Make the ratio (in fraction form) "change in *y*" to "change in *x*"

$$\frac{ychange}{xchange} = \frac{15}{3} = 5$$

We say the constant rate of change is 5.
For every change in *x* of 1 unit, there is a change in *y* of 5 units.
Every time *x* increases 1, *y* increases 5.

You find the rate of change for the descending airplane. The table has been made for you:

+___ | *x* | *y* | ___

x	*y*
10	600
50	400

Can you:

1. Find how 10 changes to get to 50
2. Find how 600 changes to get to 400
3. Make a fraction with the two changes
   a. Notice that the fraction will be negative
   b. Your fraction should be equivalent to -5

Watch Video#3 to see another example: https://www.youtube.com/watch?v=vFXXStHpfaQ&t=47s

Practice#1

Find the rate of change for these pairs of points:

2.

x	*y*
2	6
5	21

1.

x	*y*
-2	6
-5	21

4.

x	*y*
6	15
-1	29

3.

x	*y*
-4	-12
0	-8

(If you have trouble getting started on these, put the points into an x-y table)

6. (3, 7) and (10, 7)

5. $\left(\frac{1}{3}, \frac{3}{2}\right)$ and $\left(\frac{7}{3}, \frac{9}{2}\right)$

☞☞☞ Go to Schoology: do Checkpoint #1 ☜☜☜

CONSTANT RATE OF CHANGE

A ***constant*** rate of change means that the rate of change, for the particular relationship, is always the same.

It is common that you will be given a table and asked if it shows a "constant rate of change." In this case, you will need to find several rates of change and determine whether they are always the same.

Video#4 will show a few examples: https://www.youtube.com/watch?v=rmsxQlf26pw&t=340s

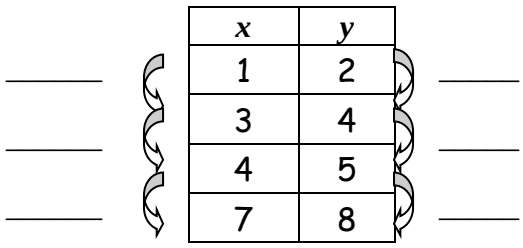
Notice in that last two examples that there was NO constant rate of change.

Sample Problem:

The table below has a new twist. See if you can work through the instructions to determine whether it shows a constant rate of change.

x	*y*
1	2
3	4
4	5
7	8

Step 1: find and write the changes in *x* and the changes in *y* between each row.



Step 2: make 3 fractions by writing each "change in *y*" OVER its partner "change in *x*"

Step 3: compare the fractions – are they all equivalent to the same fraction?

If so, the relationship has constant rate of change.

If not, the relationship does NOT have a constant rate of change.

Practice#2

Decide whether each table is showing a constant rate of change. Show how you decided.

1.

x	*y*
3	75
4	100
5	125
6	150

2.

x	*y*
4	232
6	348
8	464
10	580

3.

x	*y*
3	-9
4	-6
5	-3
6	3

4.

x	*y*
3	-5
4	0
6	5
10	10

5.

x	*y*
0	1
1	2
2	5
3	10

6.

x	*y*
1	2
3	8
6	17
7	20

☞☞☞ Go to Schoology and do Checkpoint #2 ☜☜☜

So what does a "linear relationship" have to do with all this rate of change talk?

LINEAR RELATIONSHIP DEFINITION

The definition of a linear relationship is very short, and it requires you to piece together what you have learned so far.

⟶⟶A linear relationship is one that has a **constant rate of change**. ⟵⟵

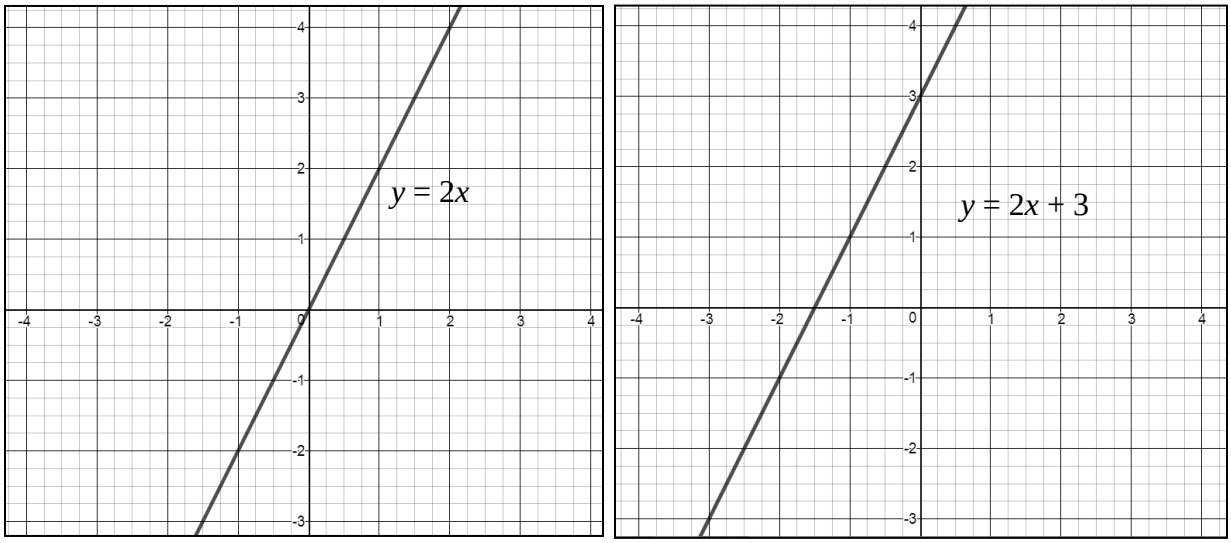
Comparing how much the **y-value changes** to how much the **x-value changes**.

In fact, in 7th grade, you already looked at some special linear relationships when you investigated proportional relationships. Keep reading to learn how 'proportional' and 'linear' relate to each other.

INTRODUCTION TO LINEAR FUNCTIONS

In 7th grade, you spent some time exploring **proportional relationships**. In 8th grade and Algebra it is very important to extend these ideas to investigating **linear relationships**.

Look at the two graphs below. You should recognize the graph on the left as being proportional. The graph on the right is **linear, but not proportional**.



These graphs and equations look similar, but not exactly alike.

Proportional vs Linear: Comparing Graphs

Both graphs are straight lines, but they appear to be in different positions.

As you already know, the proportional graph passes through the origin (0, 0). We are often interested in where a graph crosses the *y*-axis, and a proportional graph does this at $y = 0$. Notice that the linear graph does not cross the y-axis at $y = 0$; it crosses at $y = 3$.

Proportional vs Linear: Comparing Equations

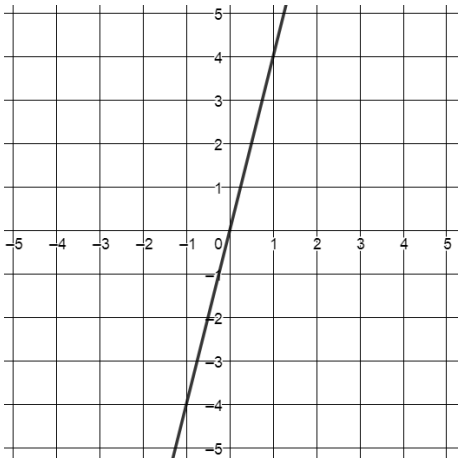
Both equations have a y and $2x$ in them, but the non-proportional graph seems to have an extra term (the "+ 3").

You have learned that the equation of every proportional relationship can be written as $y = kx$. You have called "k" the *constant of proportionality* or *unit rate*. In a linear relationship, we still are interested in what is multiplying the x, but we will be giving it new name.

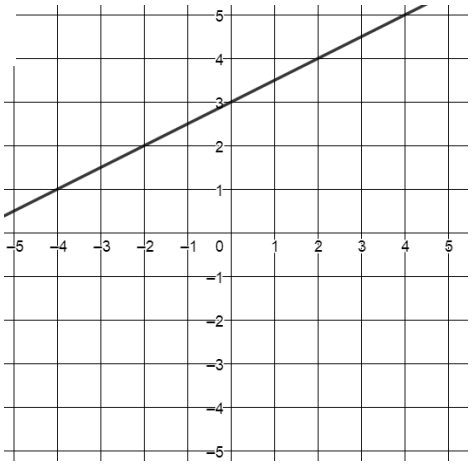
Practice#3

For each graph shown, tell whether it is Proportional, Linear or Neither. You should be able to explain how you know.

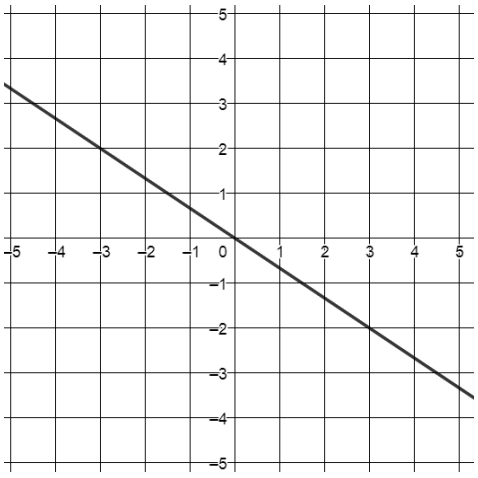
1.



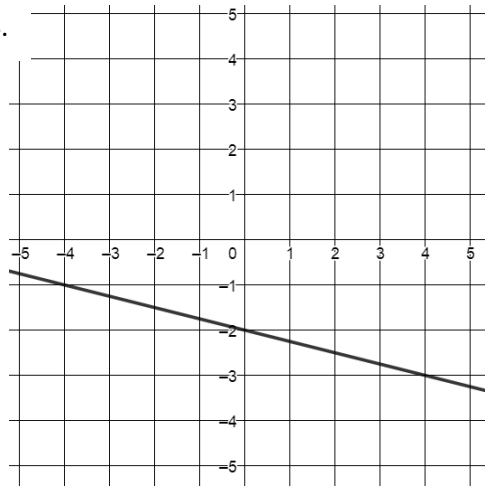
2.



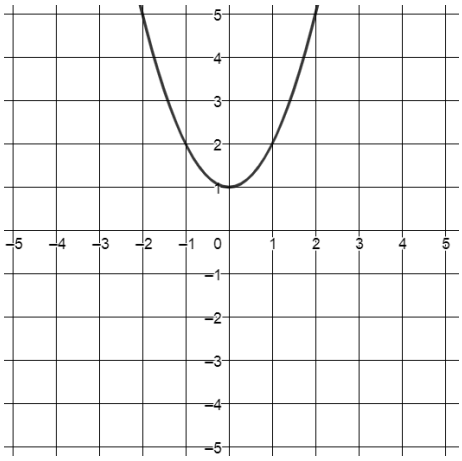
3.



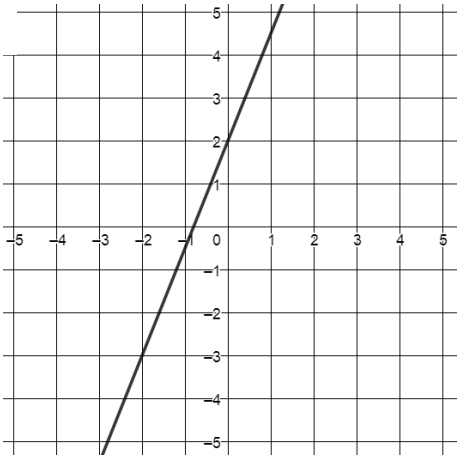
4.



5.



6.



For each equation shown, tell whether it is Proportional, Linear or Neither. You should be able to explain how you know.

7. $y = \frac{3}{5}x$

8. $y = \frac{1}{2}x - 6$

9. $y = x^2 + 3$

10. $y = -3x$

11. $y = x$

12. $y = 2 - x$

☞☞☞ Go to Schoology and do Checkpoint #3 ☜☜☜

Proportional vs Linear: Comparing Tables

This may be the trickiest comparison of the three. Examine the tables below, and the calculations shown to the side.

$y = 2x$

x	y	You have learned that for every pair (except 0,0), $\frac{y}{x}$ always = "k"
0	0	
1	2	$\frac{y}{x} = \frac{2}{1} = 2$
2	4	$\frac{y}{x} = \frac{4}{2} = 2$
3	6	$\frac{y}{x} = \frac{6}{3} = 2$
4	8	$\frac{y}{x} = \frac{8}{4} = 2$

$y = 2x + 3$

x	y	Does $\frac{y}{x}$ always = "k"?
0	3	
1	5	$\frac{y}{x} = \frac{5}{1} = 5$
2	7	$\frac{y}{x} = \frac{7}{2} = 3.5$
3	9	$\frac{y}{x} = \frac{9}{3} = 3$
4	11	$\frac{y}{x} = \frac{11}{4} = 2.25$

$\frac{y}{x}$ keeps equaling a different number, so there **isn't** a constant value of k

By now, **<u>you should understand</u>** that in relationships that are linear, but not proportional, $\frac{y}{x}$ does NOT always = the same number, but there is a ***different*** calculation that is always constant: the "rate of change."

Video#5 and Video#6 are both excellent recaps of this investigation.

https://www.youtube.com/watch?v=C7CY4l99S7M
https://www.youtube.com/watch?v=T3axRpSZY9M

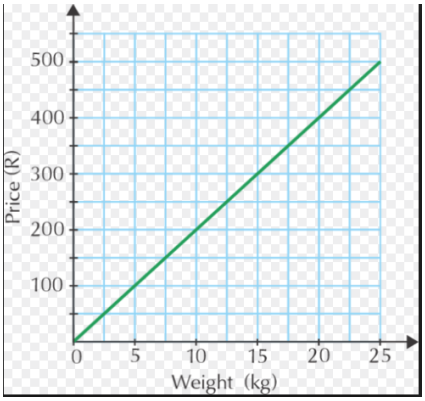
Practice#4

Decide whether each relationship shown below is proportional, linear or neither. **Be able to explain how you decided.**

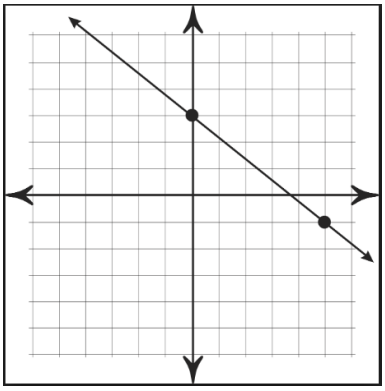
1.

x	*y*
0	0
1	1
2	4
3	9

2.



3.



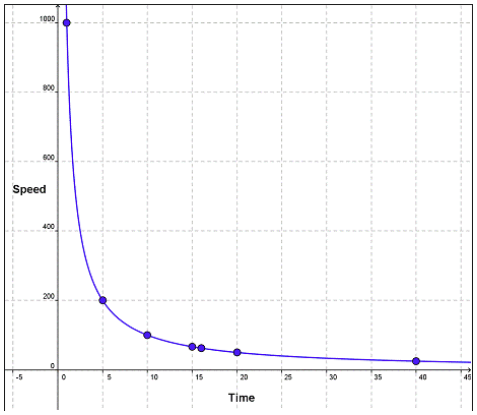
4.

x	*y*
5	-25
3	-15
2	-10
-1	5

5.

x	*y*
-2	7
0	13
2	19
4	25

6.



☞☞☞ Go to Schoology and do Checkpoint #4 ☜☜☜

Equations of Lines: Putting it all together

In previous sections, we've touched briefly on some ideas relating to linear **equations**.

At the bottom of page 10 you read this:

> In a linear relationship, we still are interested in what is multiplying the *x*, but we will be giving it new name.

And toward the bottom of page 13 you saw this:

> By now, **you should understand** that in relationships that are linear, but not proportional, $\frac{y}{x}$ does NOT always = the same number, but there is a ***different*** calculation that is always constant: the "rate of change."

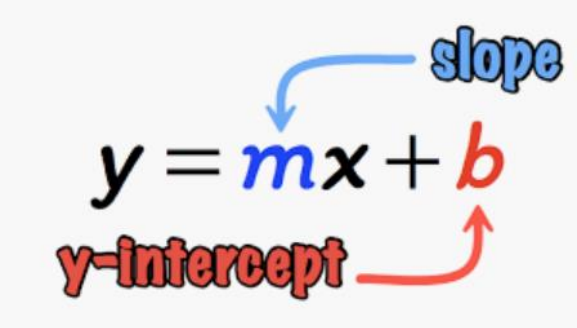
Also, in the middle of page 10:

> We are often interested in where a graph crosses the *y*-axis, ……

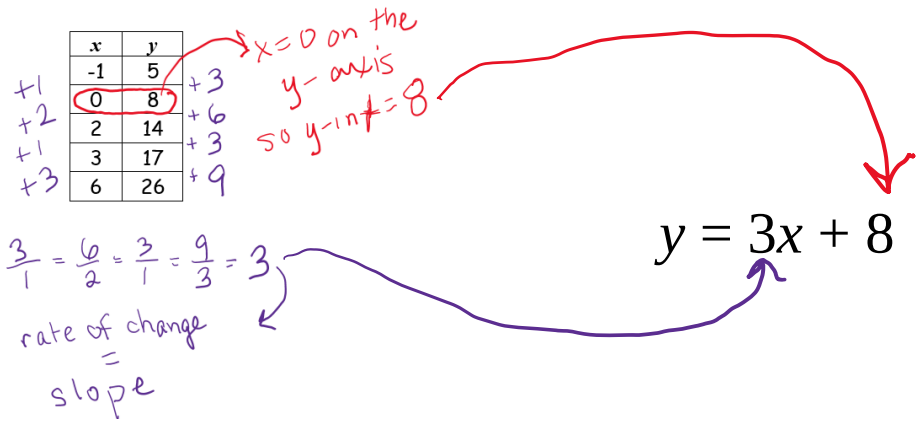
When we are talking about the line or its equation, we refer to the rate of change as "slope."

And we call the place where it crosses the *y*-axis the "*y*-intercept."

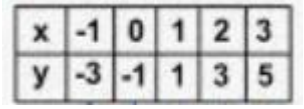
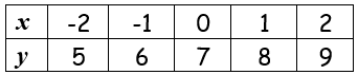
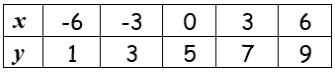
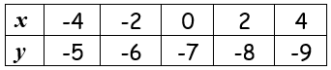
The equation for any line is often written in a standard format called "slope-intercept form."



Look at how we can make connections between the table for a linear relationship and its equation:



Look at some more examples:

table	slope	*y*-intercept	equation
	2	-1	$y = 2x - 1$
	1	7	$y = 1x + 7$ *or* $y = x + 7$
	$\frac{2}{3}$	5	$y = \frac{2}{3}x + 5$
	$-\frac{1}{2}$	-7	$y = -\frac{1}{2}x - 7$

It is important that you understand how the slope, *y*-intercept and equation are found from the table!

Practice #5

Write an equation for a line that has….

1. A slope of 4 and a *y*-intercept of 6

2. A slope of -3 and a *y*-intercept of 0.5

3. A slope of $\frac{6}{13}$ and a *y*-intercept of -0.23

4. A *y*-intercept of 0.5 and a slope of $\frac{2}{5}$

Tell the slope and *y*-intercept of these equations:

5. $y = 5x + 11$

slope = _____

y-intercept = _____

6. $y = -\frac{2}{3}x - 1$

slope = _____

y-intercept = _____

7. $y = 4 + 6x$

slope = _____

y-intercept = _____

8. $y = -3x + \frac{7}{8}$

slope = _____

y-intercept = _____

Write the equation for these tables:

9.

x	-3	-2	-1	0	1
y	12	9	6	3	0

10.

x	-6	-2	0	1
y	-13	$-6\frac{1}{3}$	-3	2

☞☞☞ Go to Schoology and do Checkpoint #5 ☜☜☜