Lesson 17: The Area of a Circle

Student Outcomes:

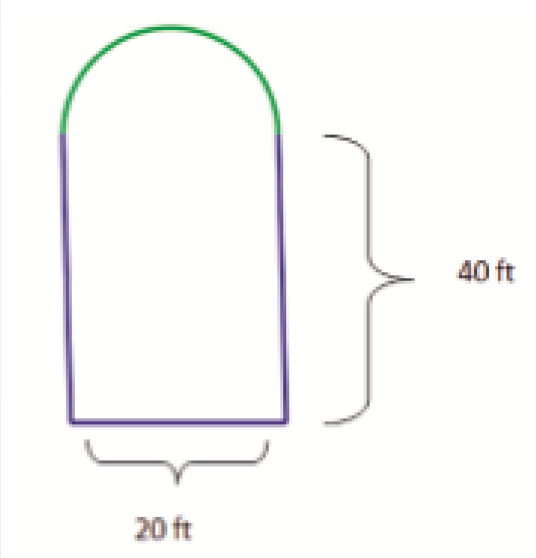
- Students give an informal derivation of the relationship between the circumference and area of a circle.
- Students know the formula for the area of a circle and use it to solve problems.

Bell Work:

Brianna's parents built a swimming pool in the backyard. Brianna says that the distance around the pool is 120 feet.

1. Is she correct? Explain why or why not.

Brianna is incorrect. The distance around the pool is 131.4 ft. She found the distance around the rectangular area only and did not include the distance around the semicircle.



2. Explain how Brianna would determine the distance around the pool so that her parents would know how many feet of stone to buy for the edging around the pool.

In order to find the distance around the pool, Brianna must first find the circumference of the semicircle, which is $C = \frac{1}{2} \bullet \pi \bullet 20\ ft.$, or 10π ft., or about 31.4 ft. The sum of the three other sides is

20 ft. + 40 ft. + 40 ft. = 100 ft.; the perimeter is

100 ft. + 31.4 ft. = 131.4 ft.

Notes:

Area of a circle:

$$A = \pi r^2$$

Solve the problem below individually. Explain your solution.

1. Find the radius of a circle if its circumference is 37.68 inches. Use $\pi \approx 3.14$.

$$37.68 = 2\pi r$$

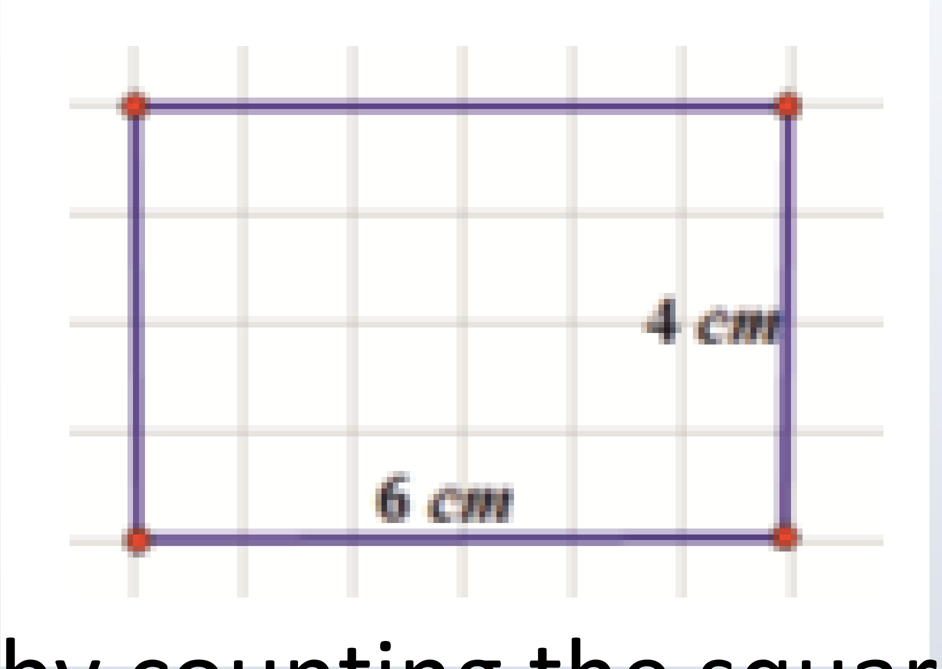
$$\left(\frac{1}{2\pi}\right)37.68 = \left(\frac{1}{2\pi}\right)2\pi r$$

$$\frac{1}{6.28}(37.68) \approx r$$

$$6 \approx r$$

The radius of the circle is approximately 6 in.

2. Determine the area of the rectangle below. Name two ways that can be used to find the area of the rectangle.



The area of the rectangle is 24 cm^2.

The area can be found by counting the square units inside the rectangle or by multiplying the length (6 cm) by the width (4 cm).

3. Find the length of a rectangle if the area is $27\ cm^2$ and the width is 3 cm.

If the area of the rectangle is
Area = length ● width, then

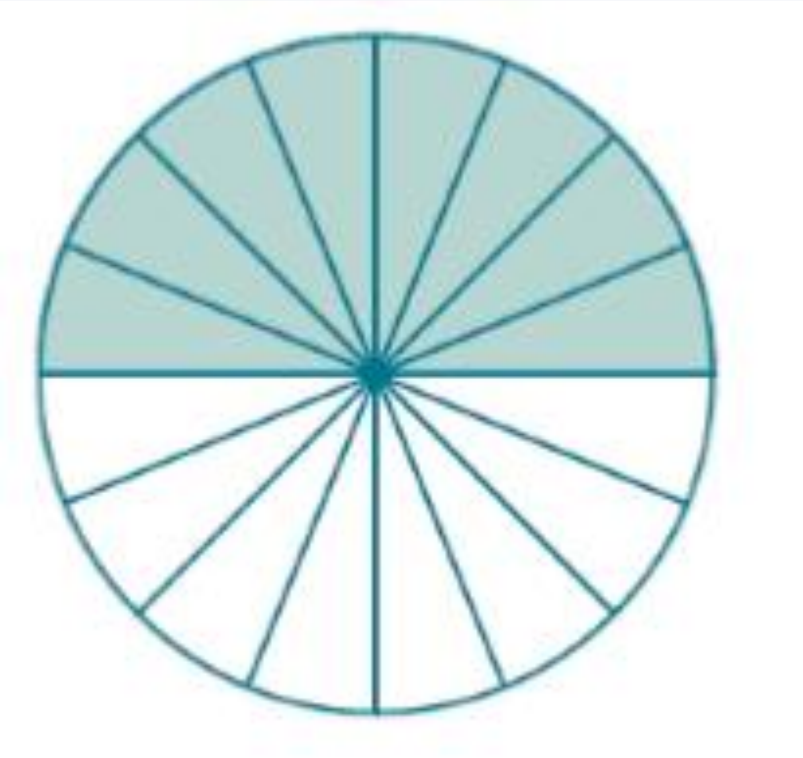
$$27\ cm^2 = l \bullet 3cm$$

$$\frac{1}{3} \bullet 27\ cm^2 = \frac{1}{3} \bullet l \bullet 3cm$$

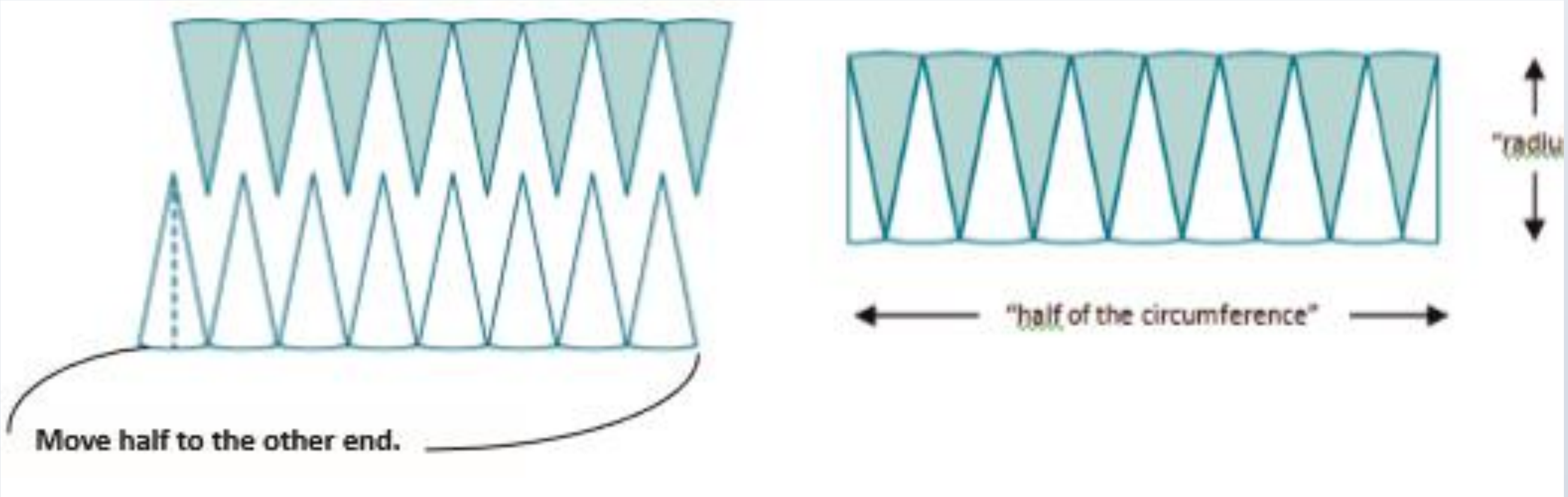
$$9cm = l$$

Exploratory Challenge: (s.112 - 113)

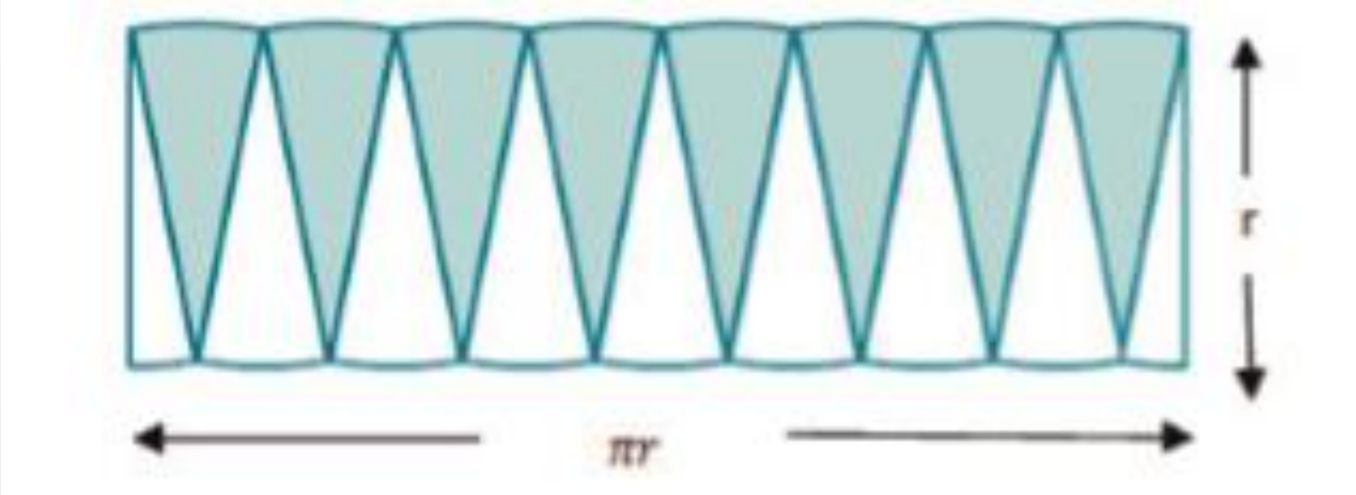
To find the formula for the area of a circle, cut a circle into 16 equal pieces.



Arrange the triangular wedges by alternating the "triangle" directions and sliding them together to make a "parallelogram." Cut the triangle on the left side in half on the given line, and slide the outside half of the triangle to the other end of the parallelogram in order to create an approximate "rectangle."



The circumference is $2\pi r$, where the radius is r.
Therefore, half of the circumference is πr.



What is the area of the “rectangle” using the side lengths above?

The area of the “rectangle” is base times height, and, in this case, A = $\pi r \bullet r$.

Are the areas of the "rectangle" and the circle the same?

Yes, since we just rearranged pieces of the circle to make the "rectangle," the area of the "rectangle" and the area of the circle are approximately equal. Note that the more sections we cut the circle into, the closer the approximation.

If the area of the rectangular shape and the circle are the same, what is the area of the circle?

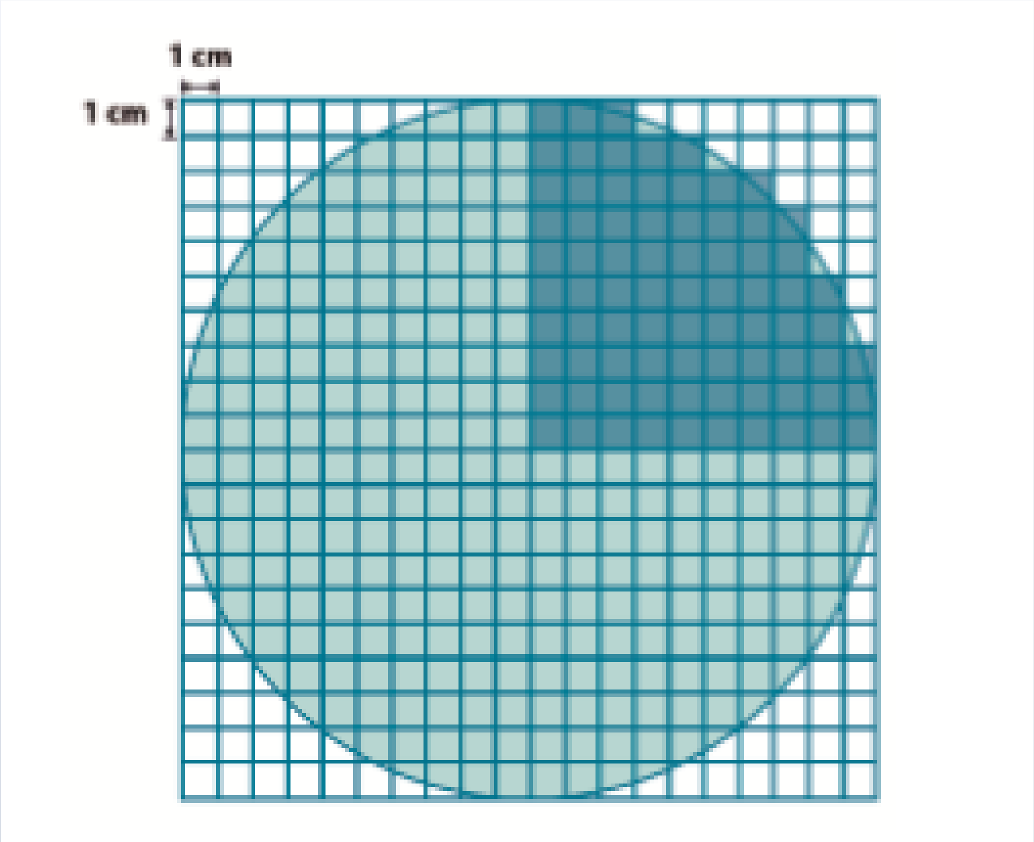
The area of a circle is written as $A = \pi r \bullet r$, or $A = \pi r^2$.

Example 1: (s 113)

Use the shaded square centimeter units to approximate the area of the circle.

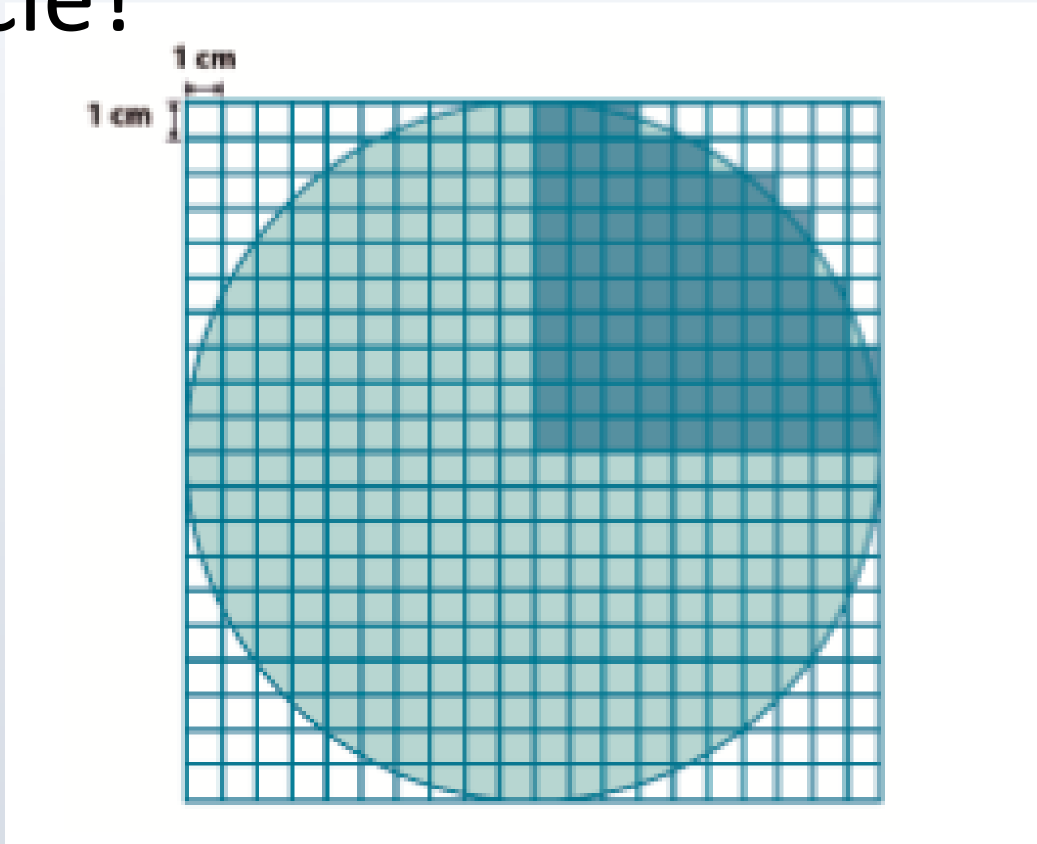
What is the radius of the circle?

10 cm



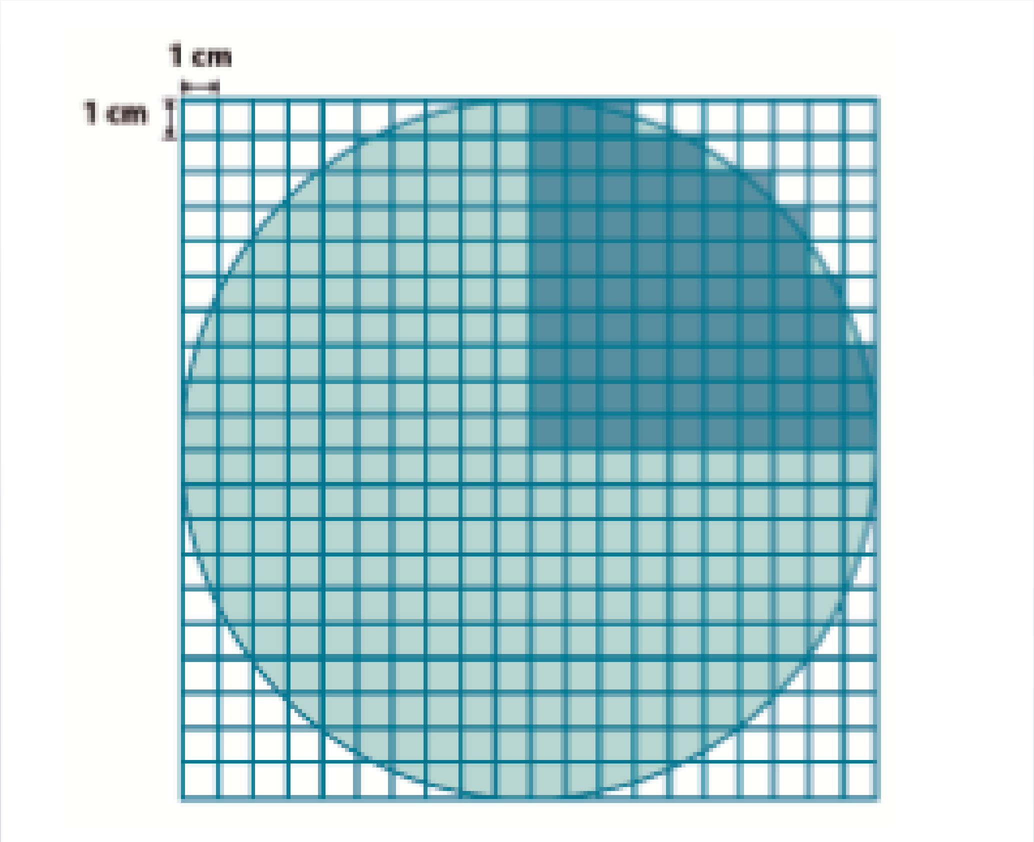
What would be a quicker method for determining the area of the circle other than counting all of the squares in the entire circle?

Count $\frac{1}{4}$ of the squares needed; then, multiply that by four in order to determine the area of the entire circle



Using the diagram, how many squares were used to cover one-fourth of the circle?

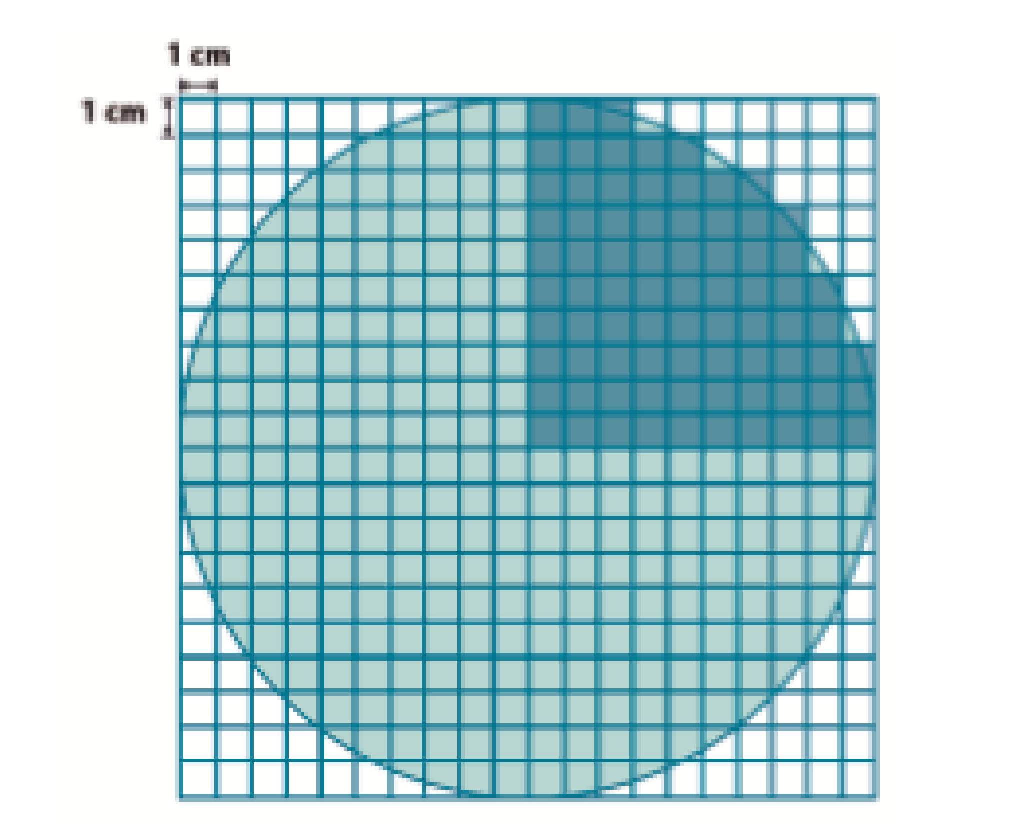
The area of one-fourth of the circle is approximately 79 cm^2.



What is the area of the entire circle?

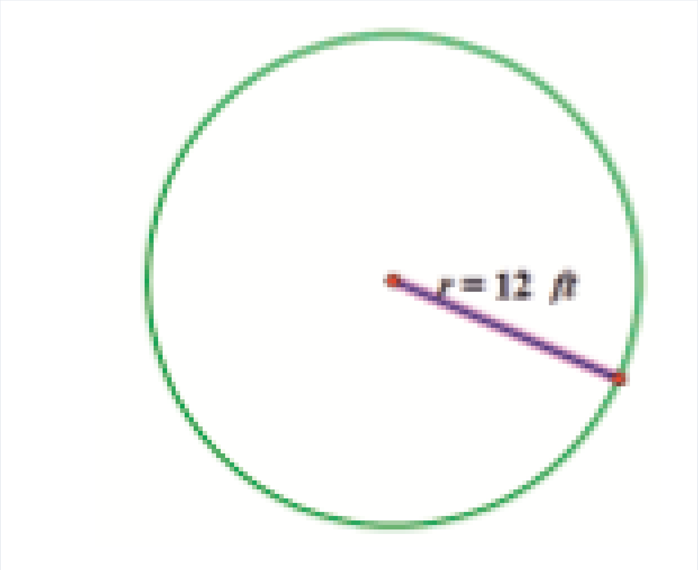
$$A \approx 4 \bullet 79\ cm^2$$

$$A \approx 316\ cm^2$$



Example 2: (s.114)

A sprinkler rotates in a circular pattern and sprays water over a distance of 12 feet. What is the area of the circular region covered by the sprinkler? Express your answer to the nearest square foot. Draw a diagram to assist you in solving the problem. What does the distance of 12 feet represent in this problem?

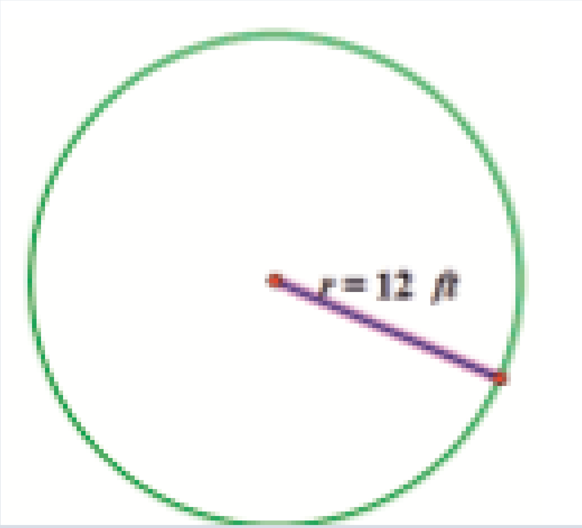


The radius is 12 feet.

What information is needed to solve the problem?

The formula to find the area of a circle is $A = \pi r^2$.

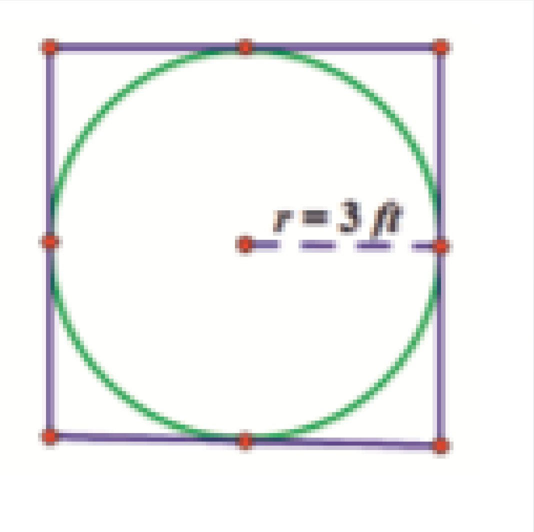
If the radius is 12 ft., then
$A = \pi \bullet (12\, ft)^2 = 144\, \pi\, ft^2$,
or approximately 452 ft².



Example 3: (s.114)

Suzanne is making a circular table out of a square piece of wood. The radius of the circle that she is cutting is 3 feet. How much waste will she have for this project? Express your answer to the nearest square foot.

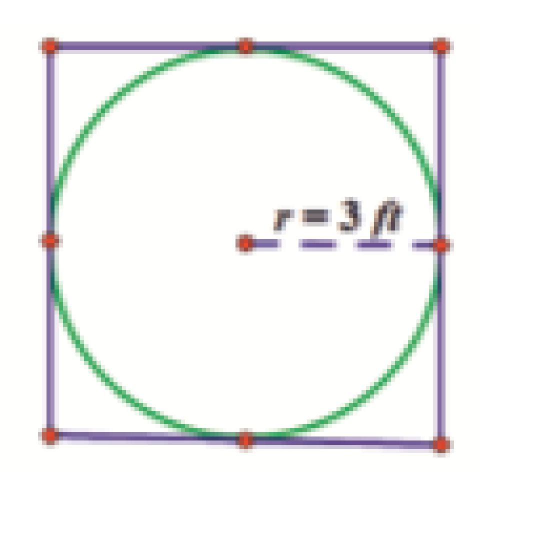
Draw a diagram to assist you in solving the problem. What does the distance of 3 feet represent in this problem?



The radius of the circle is 3 feet.

What information is needed to solve the problem?

The area of the circle and the area of the square are needed so that we can subtract the area of the circle from the area of the square to determine the amount of waste.



What information do we need to determine the area of the square and the circle?

Circle: just radius because $A = \pi r^2$

Square: one side length

How will we determine the waste?

The waste is the area left over from the square after cutting out the circular region. The area of the circle is

The area of the square is found by first finding the diameter of the circle, which is the same as the side of the square. The diameter is d = 2r; so, d = 2 ● 3 ft. or 6 ft.. The area of a square is found by multiplying the length and width, so A = 6 ft. ● 6 ft. = 36 ft^2. The solution is the difference between the area of the square and the area of the circle, so 36 ft^2 – 28.26 ft^2 $\approx$ 7.74 ft^2.

Does your solution answer the problem as stated?

Yes, the amount of waste is 7.74 ft^2.

4. A circle has a radius of 2 cm.

a. Find the exact area of the circular region.

$$A = \pi \bullet (2cm)^2 = 4\pi\ cm^2$$

b. Find the approximate area using 3.14 to approximate π.

$$A = 4cm^2 \bullet \pi \approx 4\ cm^2 \bullet 3.14 \approx 12.56\ cm^2$$

5. A circle has a radius of 7 cm.

a. Find the exact area of the circular region.

$$A = \pi \bullet (7\ cm)^2 = 49\pi\ cm^2$$

b. Find the approximate area using $\frac{22}{7}$ to approximate π.

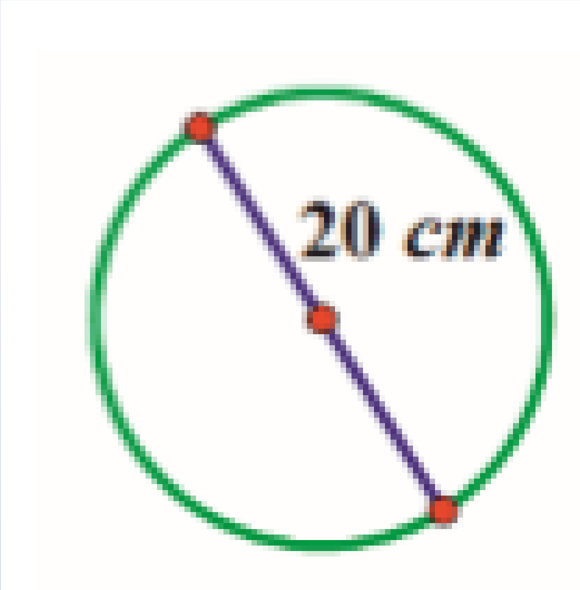
$$A = 49 \bullet \pi\ cm^2 \approx \left(49 \bullet \frac{22}{7}\right) cm^2 \approx 154 cm^2$$

c. What is the circumference of the circle?

$$C = 2\pi \bullet 7cm = 14\pi\ cm \approx 43.96\ cm$$

6. Joan determined that the area of the circle below is 400π cm^2. Melinda says that Joan's solution is incorrect; she believes that the area is 100π cm^2. Who is correct and why?

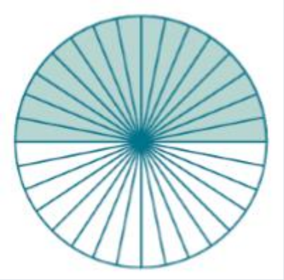
Melinda is correct. Joan found the area by multiplying π by the square of 20 cm (which is the diameter) to get a result of 400π cm^2, which is incorrect. Melinda found that the radius was 10 cm (half of the diameter). Melinda multiplied π by the square of the radius to get a result of 100π cm^2.



Closing:

Strategies for problem solving include drawing a diagram to represent the problem and identifying the given information and needed information to solve the problem.

Using the original circle in this lesson, cut it into 64 equal slices. Reassemble the figure. What do you notice?



It would look more like a rectangle.

What does the length of the rectangle become?

An approximation of half of the circumference of the circle.

What does the width of the rectangle become?

An approximation of the radius

Thus, we conclude that the area of the circle is $A = \frac{1}{2}Cr$.

- If $A = \frac{1}{2}Cr$, then $A = \frac{1}{2} \bullet 2\pi r \bullet r$ or $A = \pi r^2$.

Problem Set
(s.117)