

Lesson 16: The Most Famous Ratio of All

Student Outcomes:

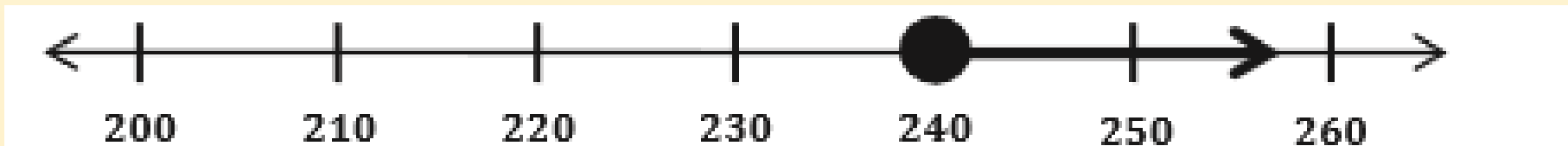
- Students develop the definition of a circle using diameter and radius.
- Students know that the distance around a circle is called the circumference and discover that the ratio of the circumference to the diameter of a circle is a special number called pi, written π .
- Students know the formula for the circumference C of a circle, of diameter d , and radius r . They use scale models to derive these formulas.
- Students use $\frac{22}{7}$ and 3.14 as estimates for π and informally show that π is slightly greater than 3.

Bell Work:

The junior high art club sells candles for a fundraiser. The first week of the fundraiser, the club sells 7 cases of candles. Each case contains 40 candles. The goal is to sell at least 13 cases. During the second week of the fundraiser, the club meets its goal. Write, solve, and graph an inequality that can be used to find the possible number of candles sold the second week.

$$\frac{n}{40} + 7 \geq 13$$

$$n \geq 240$$

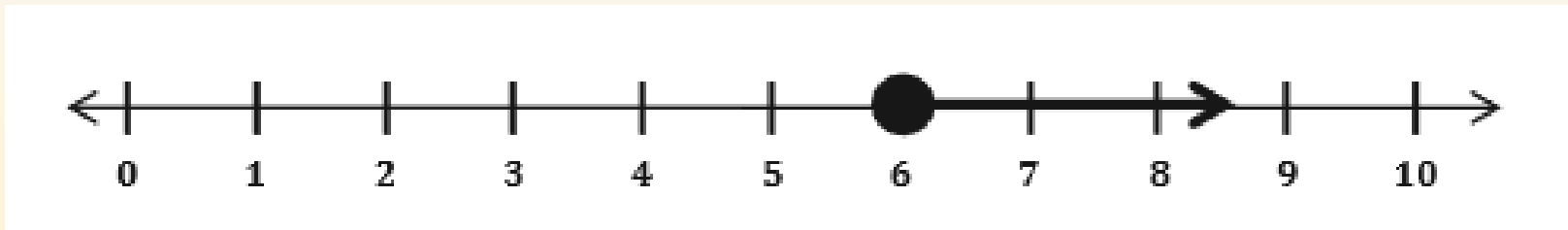


Or:

Let n represent the number of cases of candles sold the second week.

$$40n + 280 \geq 520$$

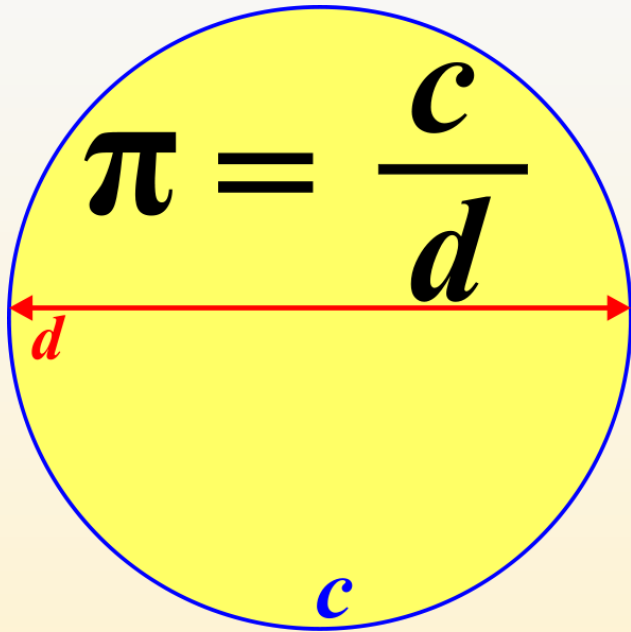
$$n \geq 6$$



The minimum number of cases sold the second week was 6. Since there are 40 candles in each case, the minimum number of candles sold the second week would be $40(6) = 240$

Notes:

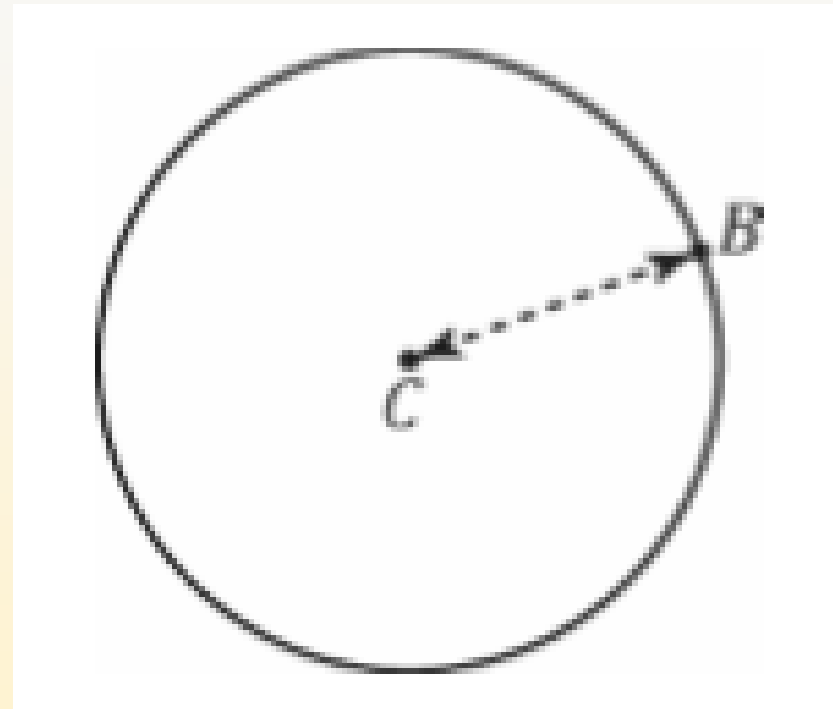
The ratio of every circle's circumference, C , to its diameter, d , is the same. It has a special symbol, π , pi.



Both 3.14 and $\frac{22}{7}$ are good approximations for this ratio. Use $\frac{22}{7}$ for π when calculations involve fractions and use 3.14 when they do not.

The most accurate measurement of all is π .

- a. Using a compass, draw a circle like the picture to the right.
- C is the center of the circle.
 - The distance between C and B is the radius of the circle.



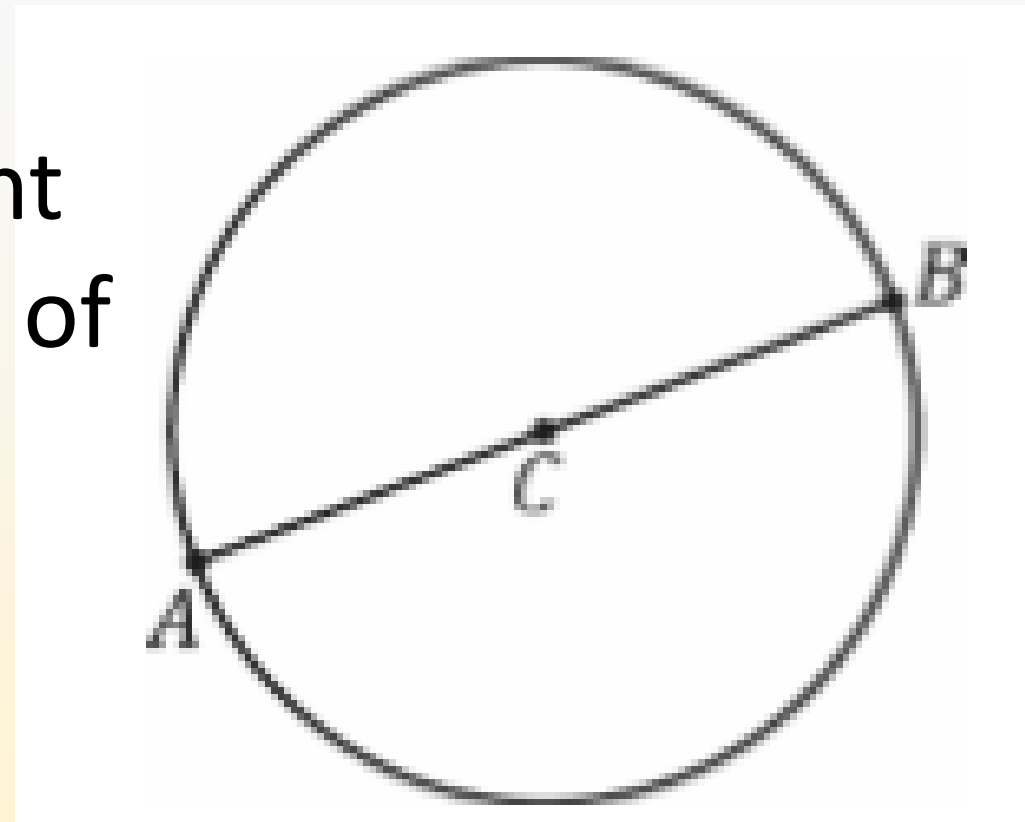
b. Write your own definition for the term circle.

Given a point O in the plane and a number $r > 0$, the circle with center O and a radius r is the set of all points in the plane whose distance from the point O is equal to r .

c. Extend segment CB to a segment AB in part A, where A is also a point on the circle.

The length of the segment AB is called the diameter of the circle.

d. The diameter is twice, or 2 times, as long as the radius.



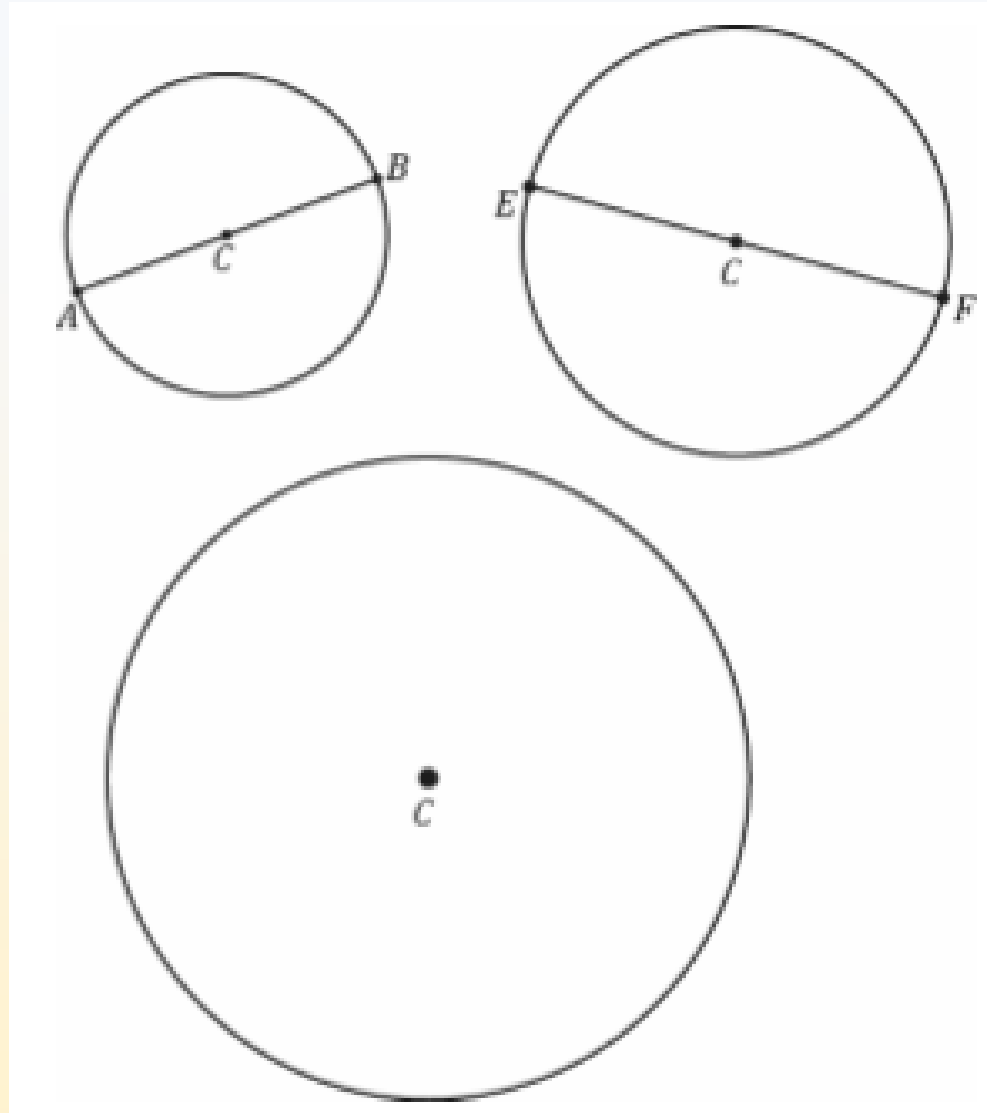
e. Measure the radius and diameter of each circle. The center of each circle is labeled C.

$$CB = 1.5 \text{ cm}; AB = 3 \text{ cm}$$

$$CF = 2 \text{ cm}; EF = 4 \text{ cm}$$

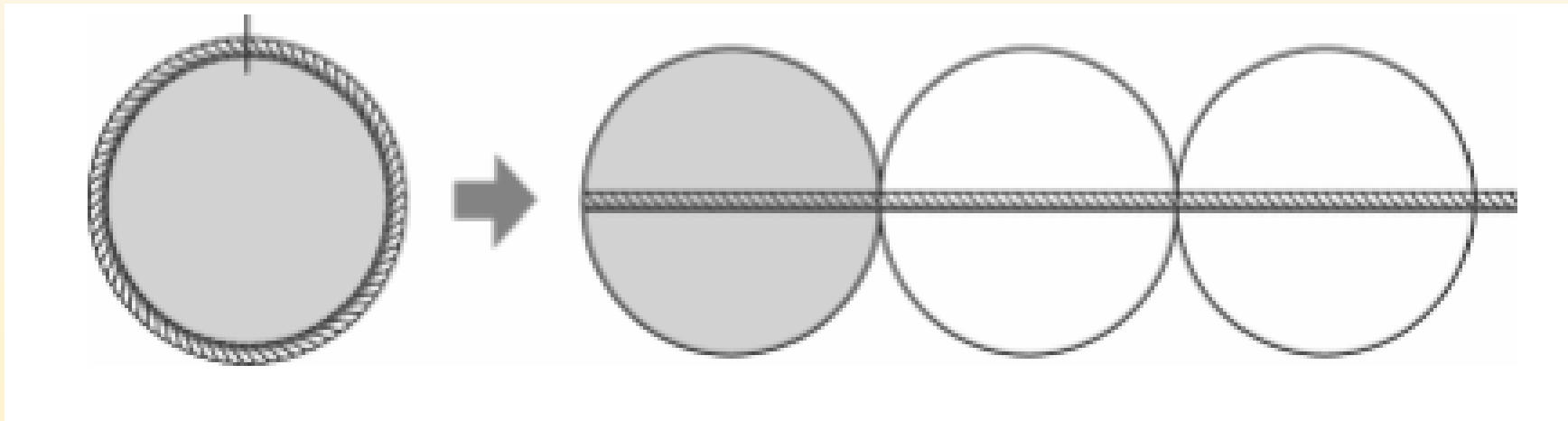
The radius of the largest circle is 3 cm .

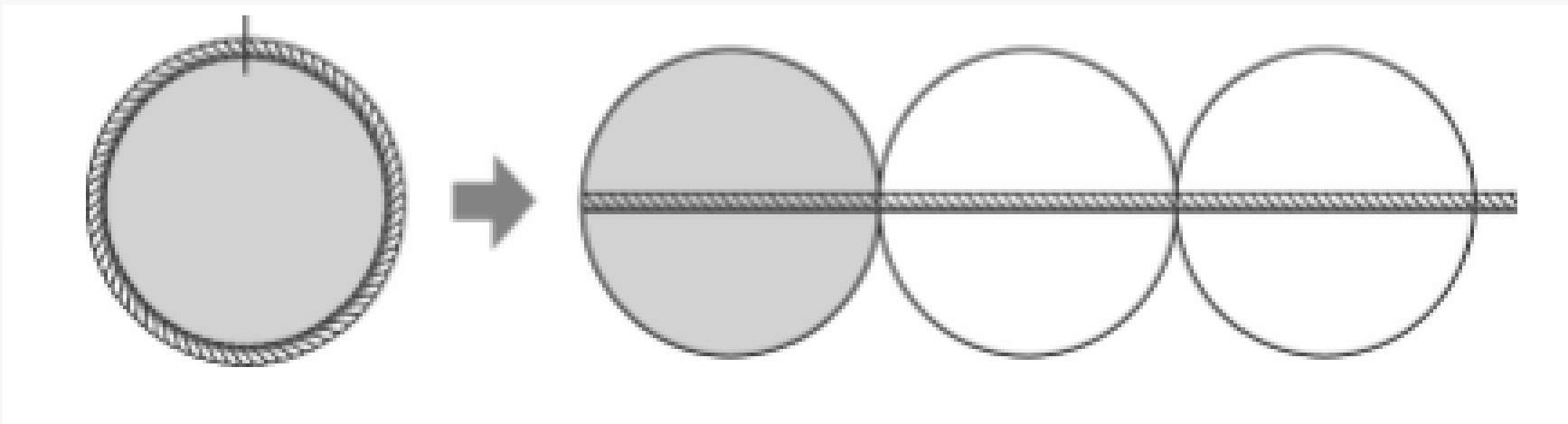
The diameter is 6 cm.



f. Draw a circle of radius 6 cm.

The ratio of the circumference to its diameter is always the same for any circle. The value of this ratio, $\frac{\textit{Circumference}}{\textit{Diameter}}$, is called the number pi and is represented by the symbol π .



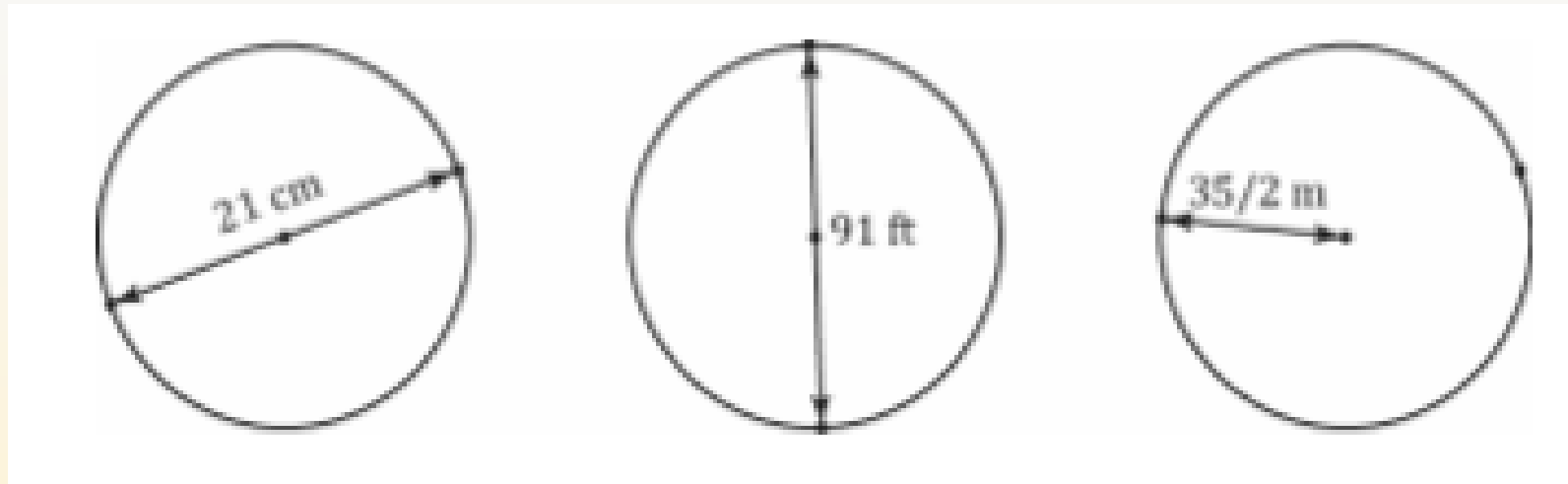


Since the circumference is a little greater than 3 times the diameter, π is a number that is a little greater than 3. Use the symbol π to represent this special number. Pi is a non-terminating, non-repeating decimal, and mathematicians use the symbol π or approximate representations as more convenient ways to represent pi.

- $\pi \approx 3.14$ or $\frac{22}{7}$.
- The ratios of the circumference to the diameter and $\pi : 1$ are equal
- Circumference of a Circle = $\pi \times \text{Diameter}$.

Example: (s.105)

a. The following circles are not drawn to scale. Find the circumference of each circle. (Use $\frac{22}{7}$ as an approximation for π .)



66 cm

286 ft.

110 m

Are these numbers roughly three times the diameters?

b. The radius of a paper plate is 11.7 cm. Find the circumference to the nearest tenth. (Use 3.14 as an approximation for π .)

Diameter: 23.4 cm

Circumference: 73.5 cm

c. The radius of a paper plate is 11.7 cm. Find the circumference to the nearest hundredth. (Use the π button on your calculator as an approximation for π .)

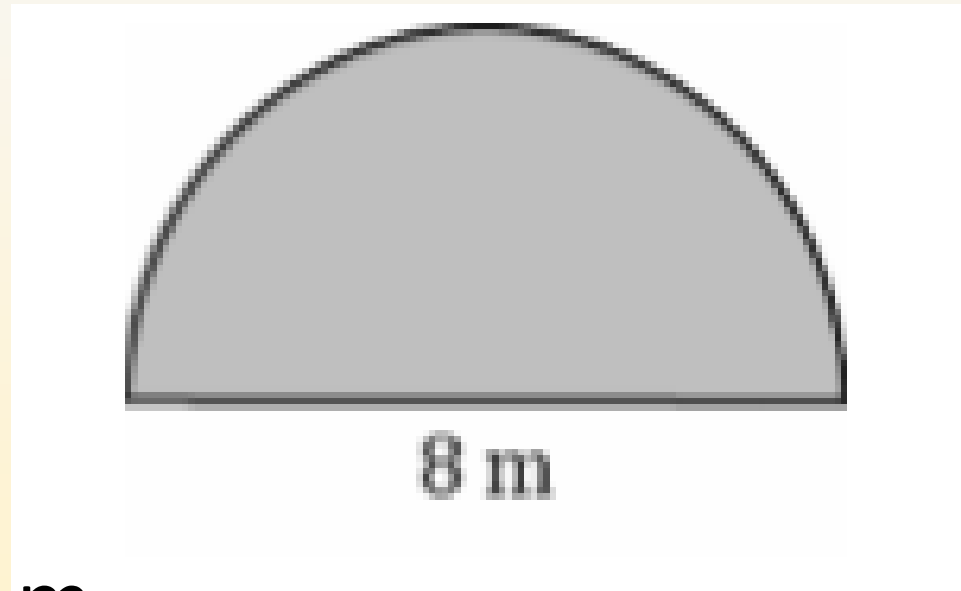
Circumference: 73.51 cm

d. A circle has a radius of r cm and a circumference of C cm. Write a formula that expresses the value of C in terms of r and π .

$$C = \pi \bullet 2r$$

$$C = 2\pi r$$

e. The figure below is in the shape of a semicircle. A semicircle is an arc that is half of a circle. Find the perimeter of the shape. (Use 3.14 for π .)



$$8m + \frac{8(3.14)}{2}m = 20.56 m$$

Closing:

CIRCLE: Given a point O in the plane and a number $r > 1$, the circle with center O and radius r is the set of all points in the plane whose distance from the point O is equal to r .

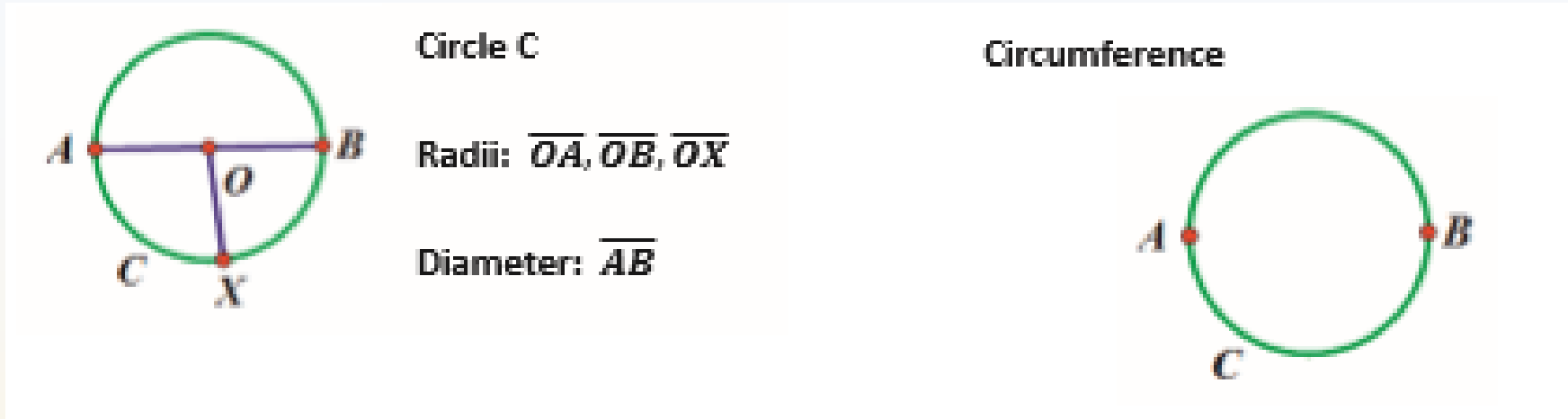
RADIUS OF A CIRCLE: The radius is the length of any segment whose endpoints are the center of a circle and a point that lies on the circle.

Closing:

DIAMETER OF A CIRCLE: The diameter of a circle is the length of any segment that passes through the center of a circle whose endpoints lie on the circle. If r is the radius of a circle, then the diameter is $2r$.

The word diameter can also mean the segment itself. Context determines how the term is being used: The diameter usually refers to the length of the segment, while a diameter usually refers to a segment. Similarly, a radius can refer to a segment from the center of a circle to a point on the circle.

Closing:



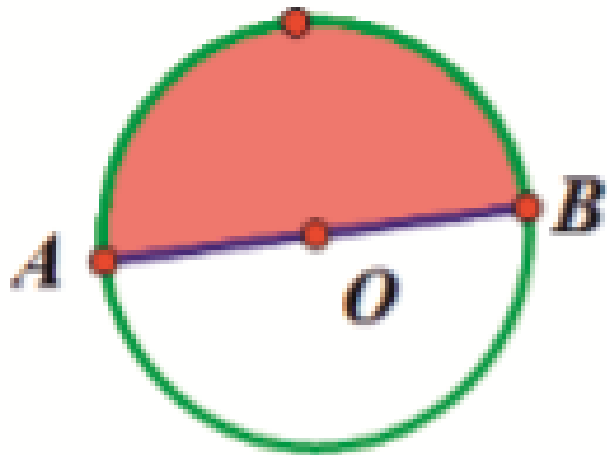
CIRCUMFERENCE: The circumference of a circle is the distance around a circle.

Closing:

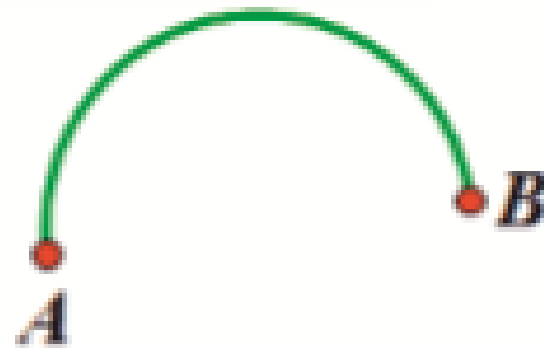
PI: The number pi, denoted by π , is the value of the ratio given by the circumference to the diameter, that is $\pi = \frac{\textit{circumference}}{\textit{diameter}}$. The most commonly used approximations for π is 3.14 or $\frac{22}{7}$.

Closing:

SEMICIRCLE: Let C be a circle with center O , and let A and B be the endpoints of a diameter. A semicircle is the set containing A , B , and all points that lie in a given half-plane determined by \overline{AB} (diameter) that lie on circle C .



Semicircle



Problem Set:
(s.108-109)