

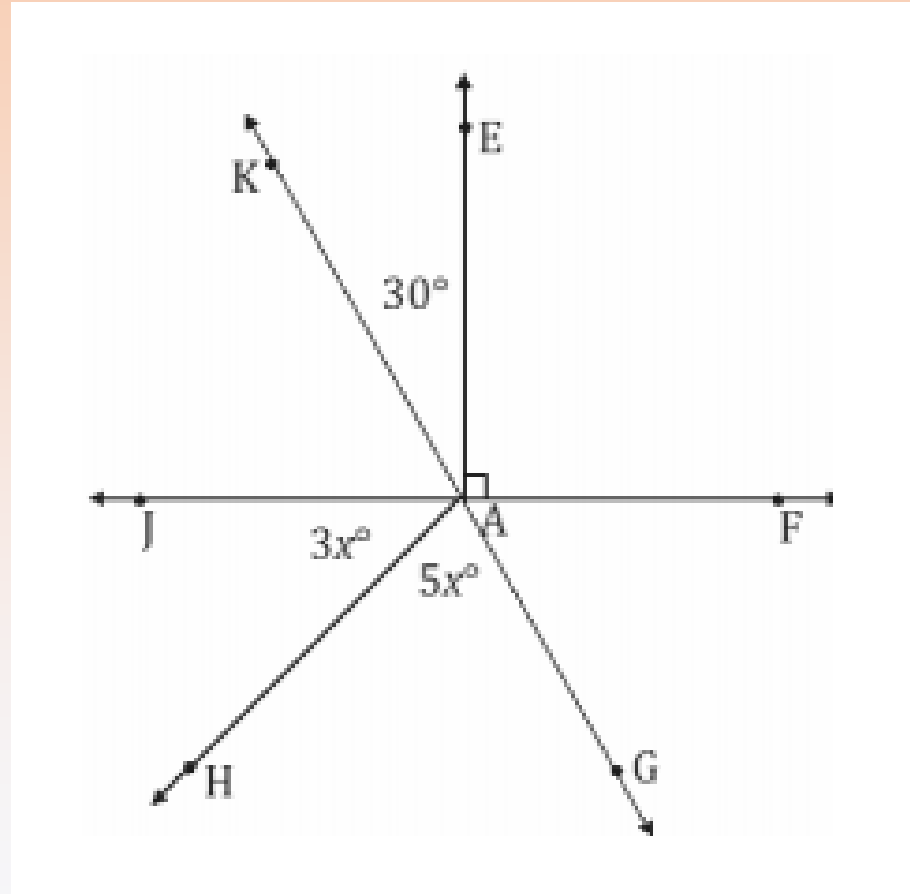
Lesson 11: Angle Problems and Solving Equations

Student Outcomes:

- Students use vertical angles, adjacent angles, angles on a line, and angles at a point in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

Bell work:

Describe the angle relationship in the diagram, then write an equation to solve for x . Determine the measurement of $\angle JAH$ and $\angle HAG$.



Notes:

Complementary angles – two or more angles that have a sum of 90° .

Supplementary angles – two or more angles that have a sum of 180° .

Adjacent angles – share a vertex and a side but no points in their interiors.

Vertical angles are formed by two intersecting lines and are opposite each other. Vertical angles have the same measure.

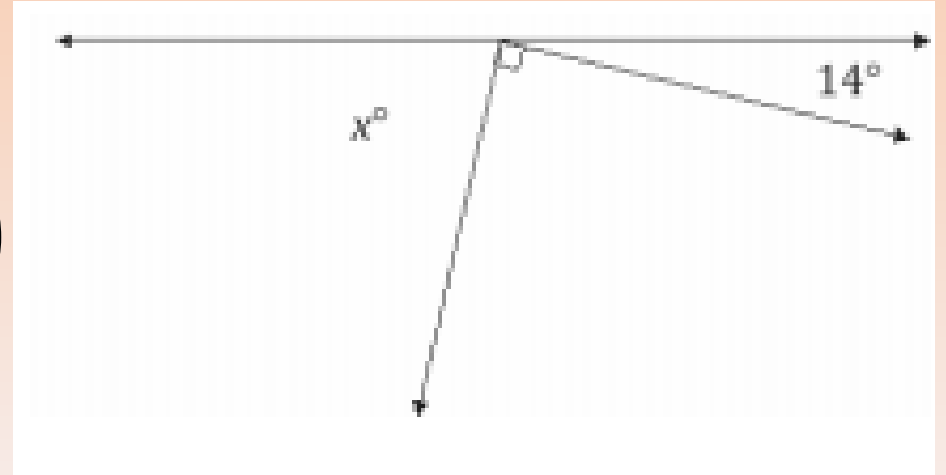
a. Angles on a line and have a sum of 180°

$$x + 90 + 14 = 180$$

$$x + 104 = 180$$

$$x + 104 - 104 = 180 - 104$$

$$x = 76$$

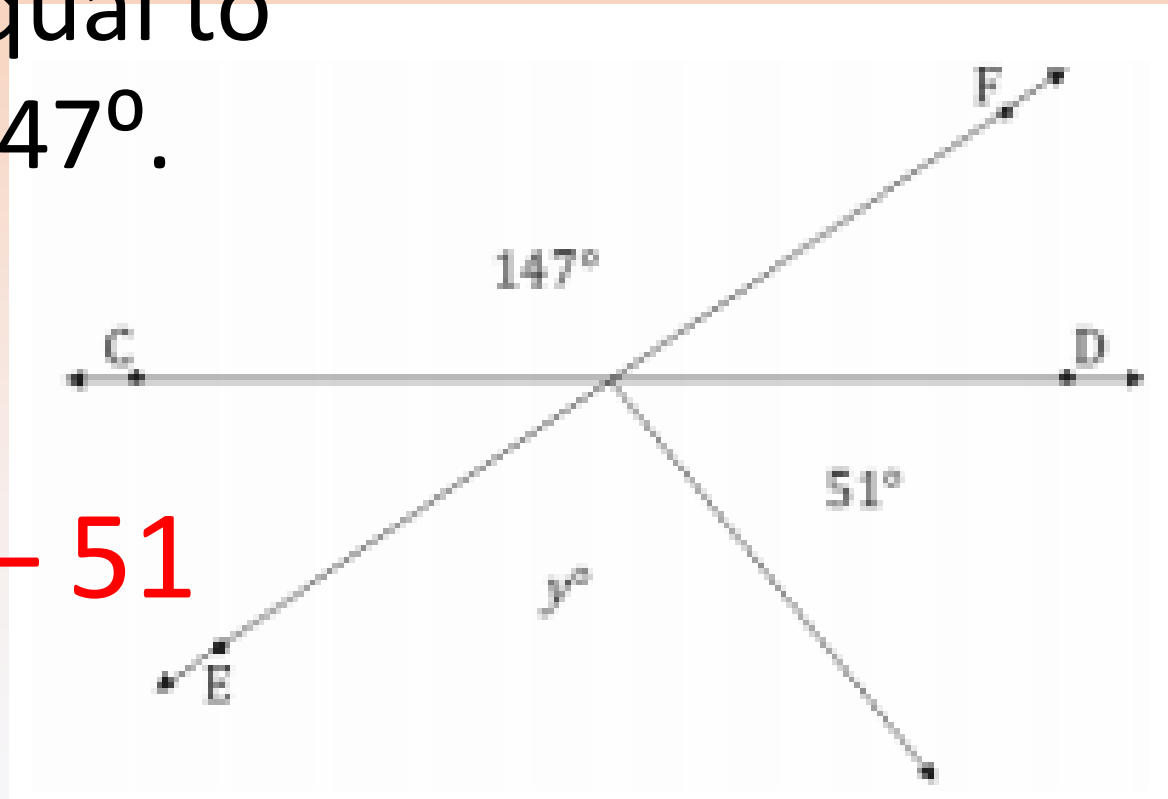


b. Adjacent angles marked by y° and 51° form an angle that is opposite and equal to the angle measuring 147° .

$$y + 51 = 147$$

$$y + 51 - 51 = 147 - 51$$

$$y = 96$$



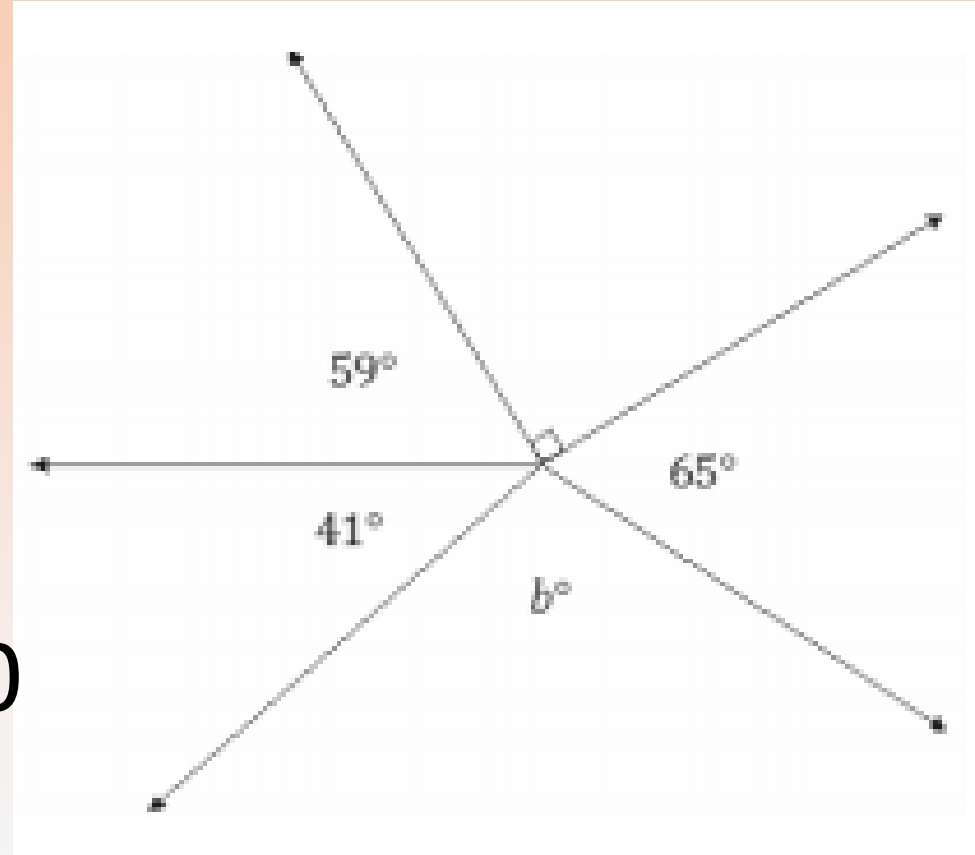
c. The adjacent angles marked by 59° , 41° , b° , 65° , and 90° are angles at a point and they have a sum of 180° .

$$59 + 41 + b + 65 + 90 = 360$$

$$b + 255 = 360$$

$$b + 255 - 255 = 360 - 255$$

$$b = 105$$



d. The angles marked by z° , 158° , and z° are angles on a line and have a sum of 180° .

$$z + 158 + z = 180$$

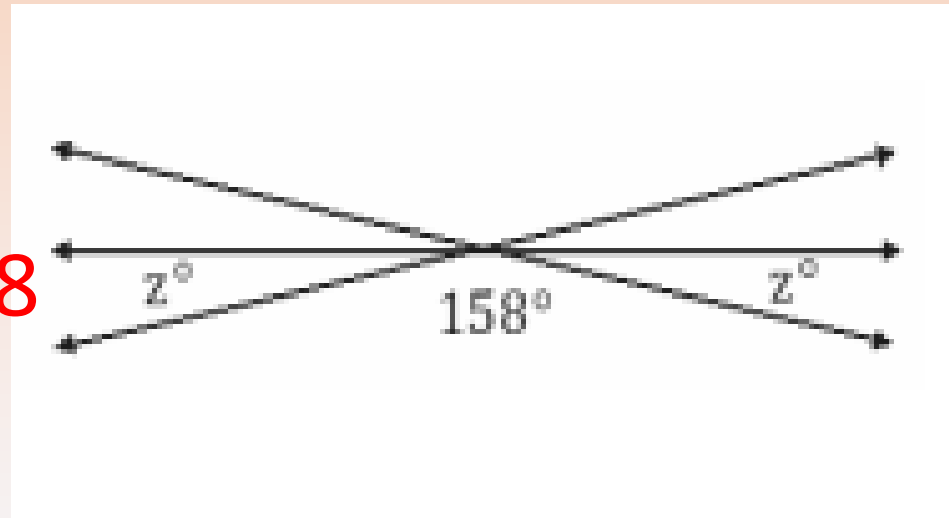
$$2z + 158 = 180$$

$$2z + 158 - 158 = 180 - 158$$

$$2z = 22$$

$$\left(\frac{1}{2}\right) 2z = \left(\frac{1}{2}\right) 22$$

$$z = 11$$



e. $\angle CPA$, $\angle CPE$, and $\angle EPB$ are angles on a line and their measures have a sum of 180° .

$$5x + 90 + x = 180$$

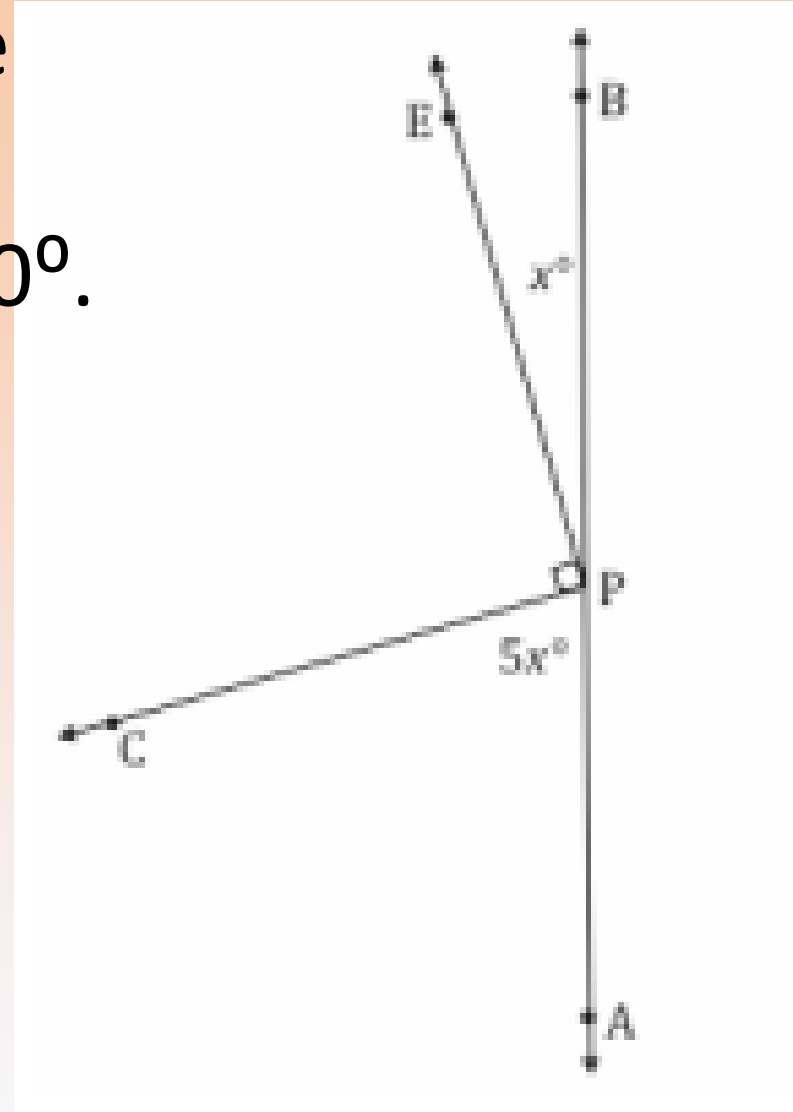
$$6x + 90 = 180$$

$$6x + 90 - 90 = 180 - 90$$

$$6x = 90$$

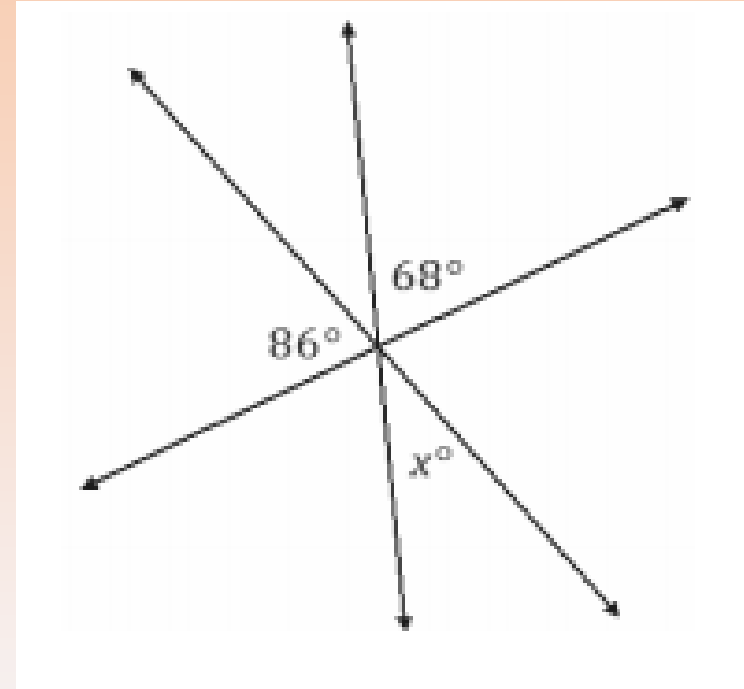
$$\left(\frac{1}{6}\right) 6x = \left(\frac{1}{6}\right) 90$$

$$x = 15 \quad \angle EPB = 15^\circ \quad \& \quad \angle CPA = 5(15) = 75^\circ$$



Example 1: (s.75)

The following figure shows three lines intersecting at a point. In a complete sentence, describe the angle relationship in the diagram. Write an equation for the angle relationship shown in the figure and solve for x .



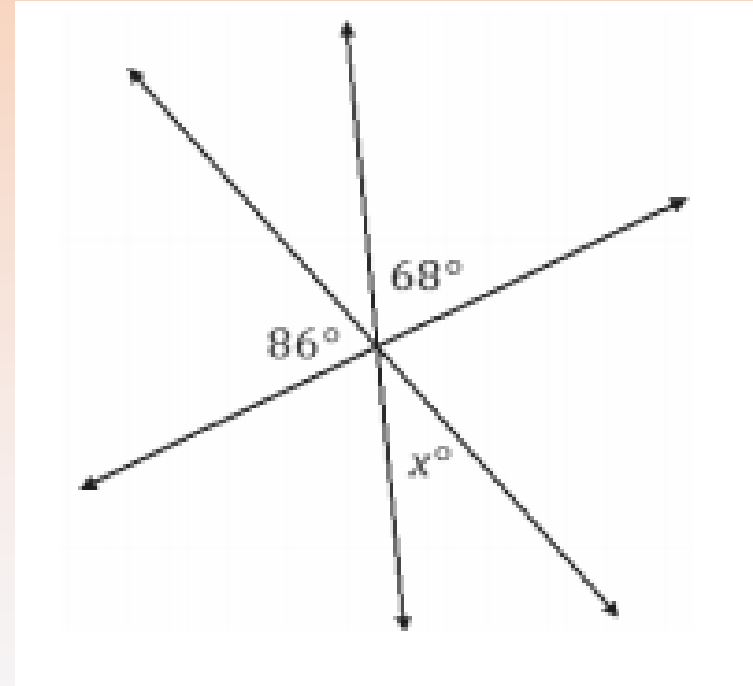
The angles 86° , 68° , and the angle between them, which is vertically opposite and equal in measure to x , are angles on a line and have a sum of 180° .

$$86 + x + 68 = 180$$

$$x + 154 = 180$$

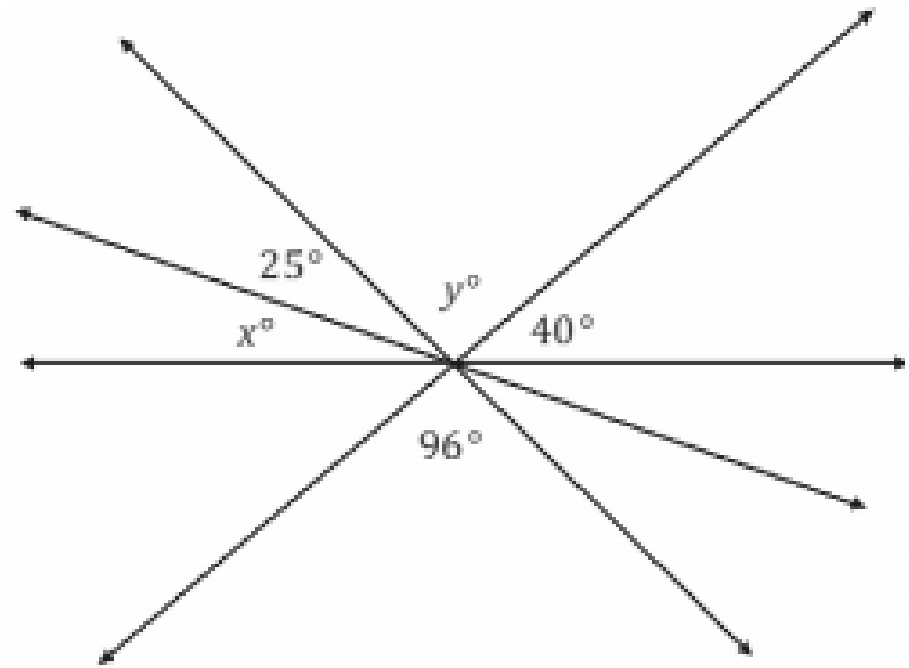
$$x + 154 - 154 = 180 - 154$$

$$x = 26$$



Exercise 1: (s.75)

The following figure shows four lines intersecting at a point. In a complete sentence, describe the angle relationship in the diagram. Write an equation for the angle relationship shown in the figure and solve for x and y .



The angles x° , 25° , y° , and 40° are angles on a line and have a sum of 180° ; the angle marked y° is vertically opposite and equal to 96° .

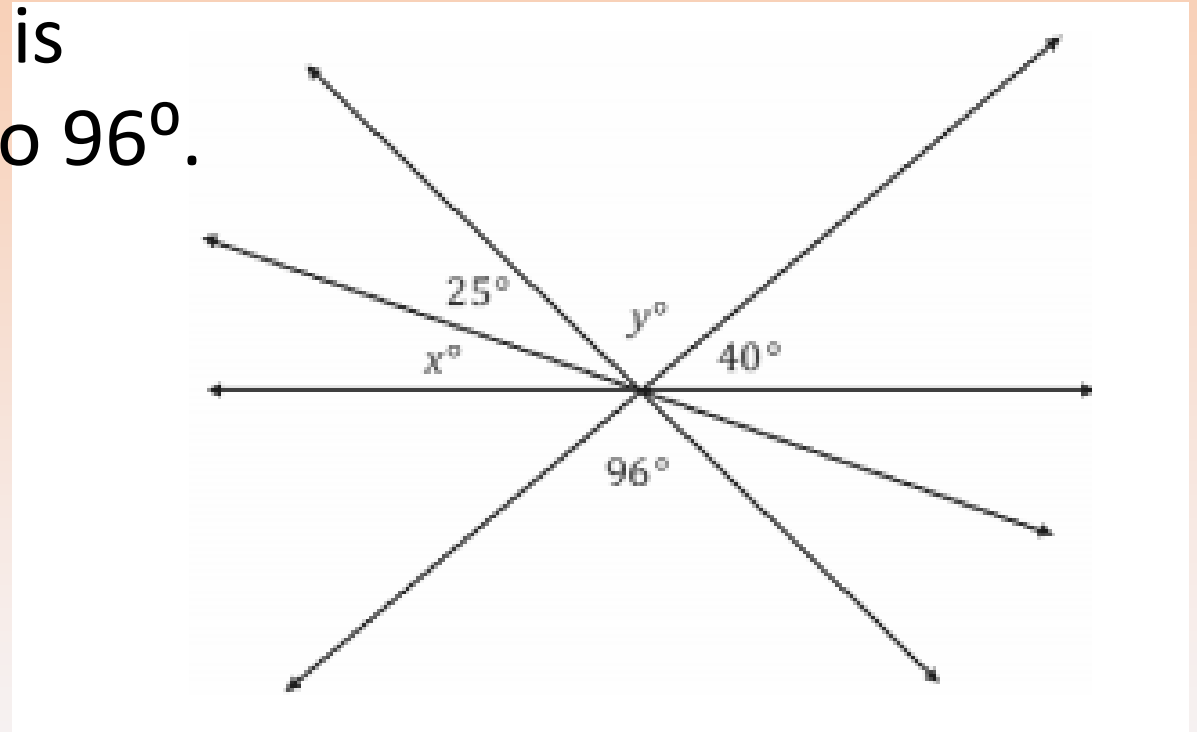
$$y = 96, \text{ vert. } \angle s$$

$$x + 25 + 96 + 40 = 180$$

$$x + 161 = 180$$

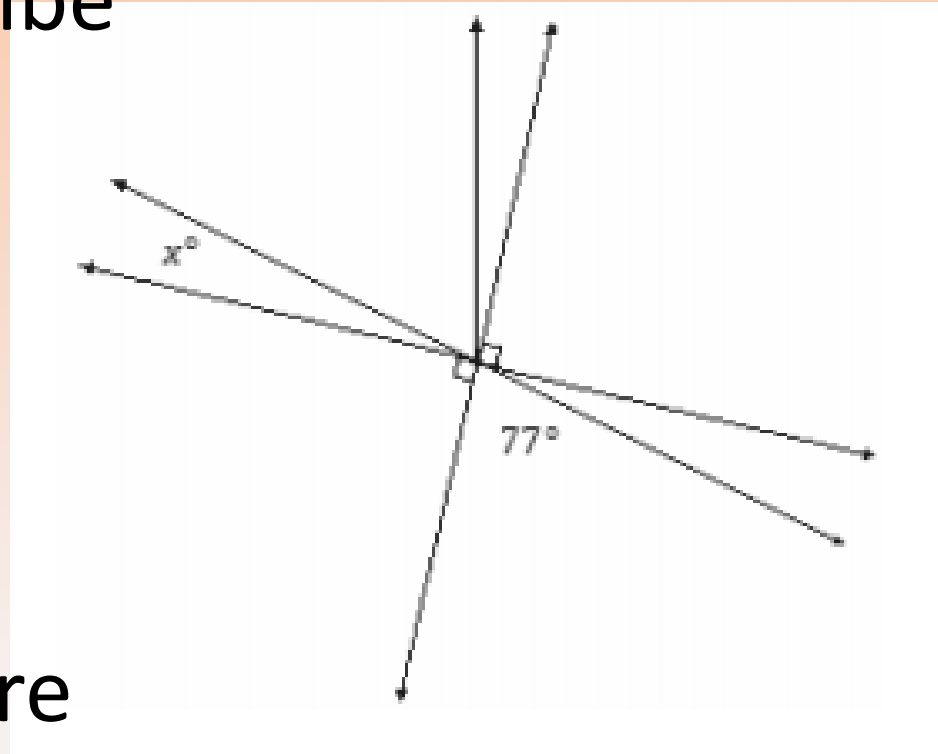
$$x + 161 - 161 = 180 - 161$$

$$x = 19$$



Example 2: (s.75)

In a complete sentence, describe the angle relationships in the diagram. You may label the diagram to help describe the angle relationships. Write an equation for the angle relationship shown in the figure and solve for x .

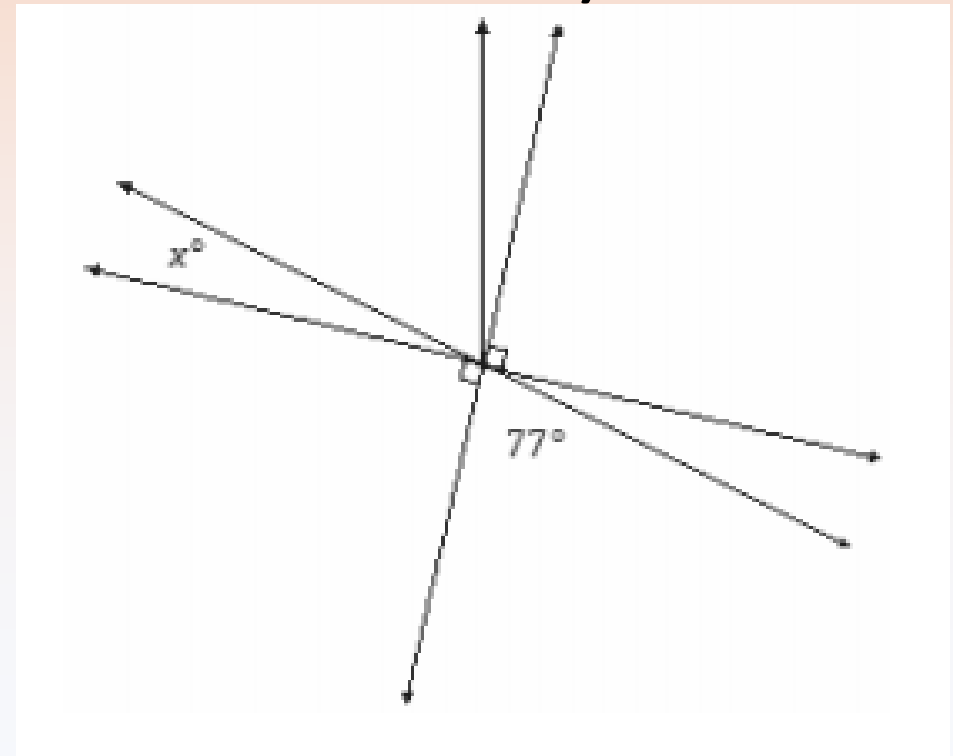


The angle formed by adjacent angles a° and b° is vertically opposite to the 77° angle. The angles x° , a° , and b° are adjacent angles that have a sum of 90° (since the adjacent angle is a right angle and together the angles are on a line).

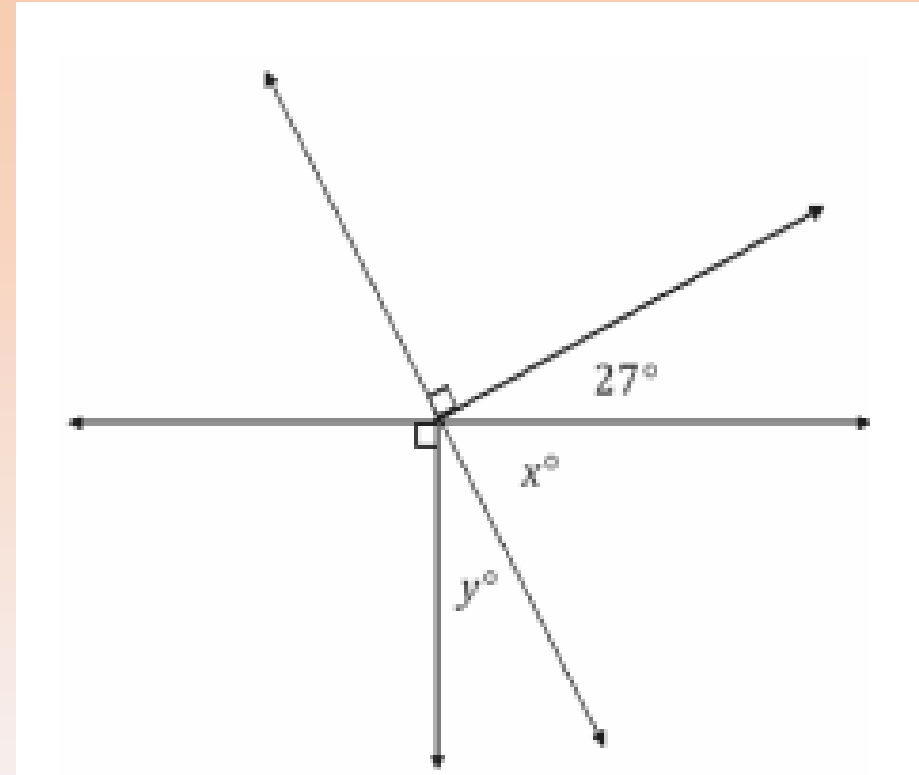
$$x + 77 = 90$$

$$x + 77 - 77 = 90 - 77$$

$$x = 13$$



In a complete sentence, describe the angle relationships in the diagram. Write an equation for the angle relationship shown in the figure and solve for x and y .



The measures of angles x and y have a sum of 90° ; the measure of angle x and 27 have a sum of 90° .

$$x + 27 = 90$$

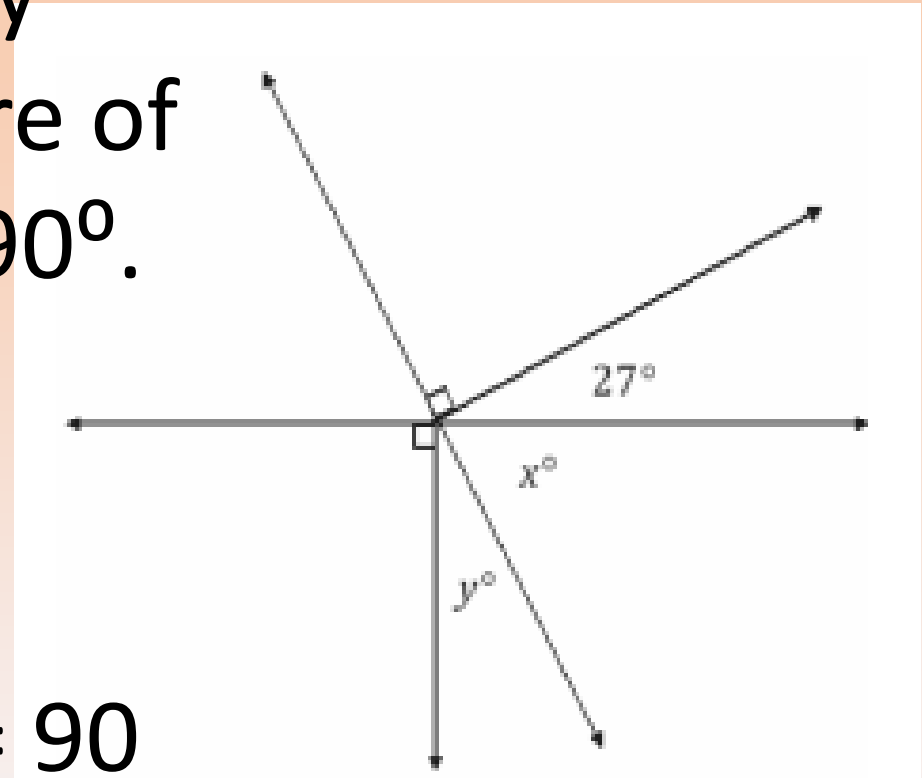
$$x + 27 - 27 = 90 - 27$$

$$x = 63$$

$$63 + y = 90$$

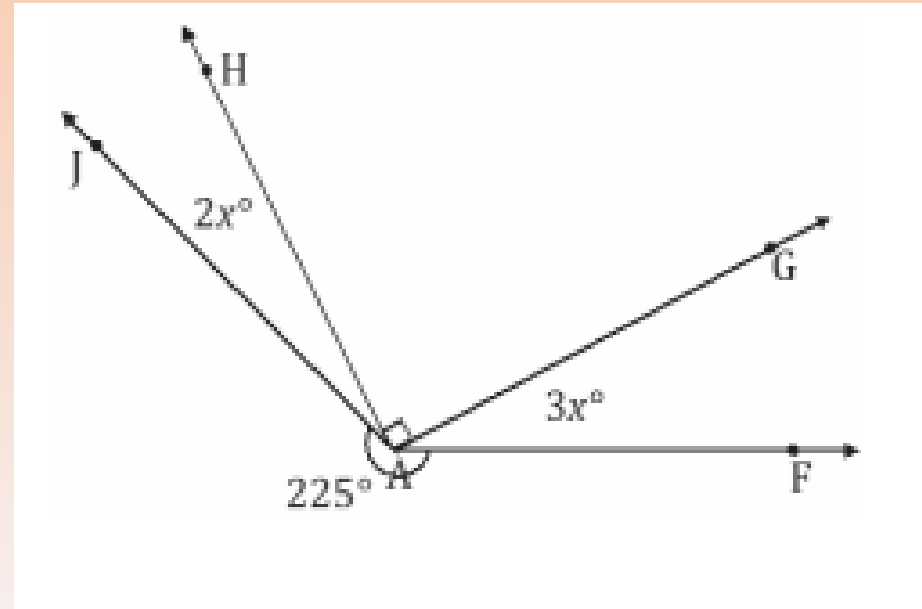
$$63 - 63 + y = 90 - 63$$

$$y = 27$$



Example 3: (s76)

In a complete sentence, describe the angle relationship in the diagram. Write an equation for the angle relationship shown in the figure and solve for x . Find the measure of $\angle JAH$ and $\angle GAF$.



The sum of the degree measurements of $\angle JAH$, $\angle GAH$ and $\angle GAF$, and the arc that subtends $\angle JAF$ is 360° .

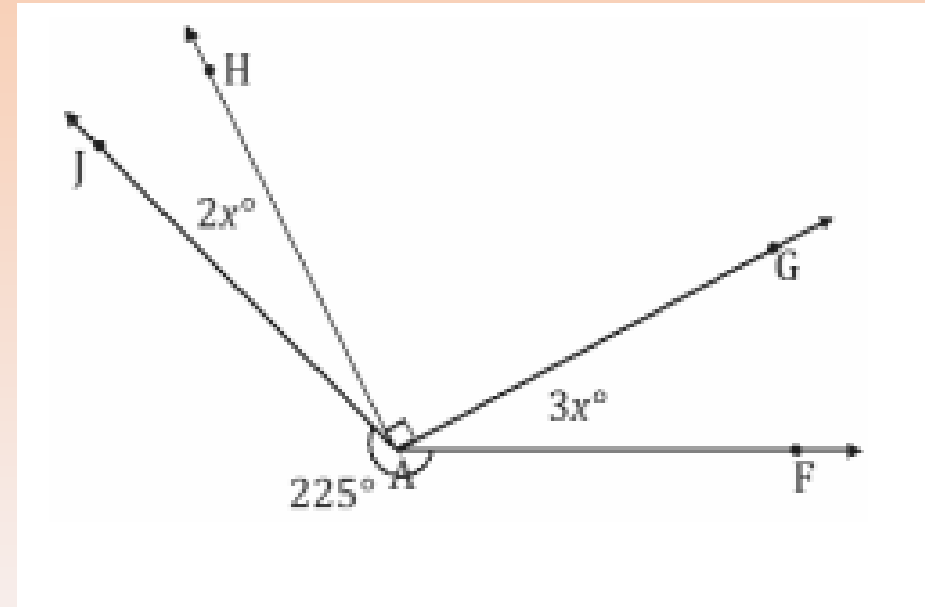
$$225 + 2x + 90 + 3x = 360$$

$$315 + 5x = 360$$

$$315 - 315 + 5x = 360 - 315$$

$$5x = 45$$

$$\left(\frac{1}{5}\right) 5x = \left(\frac{1}{5}\right) 45 \quad x = 9$$

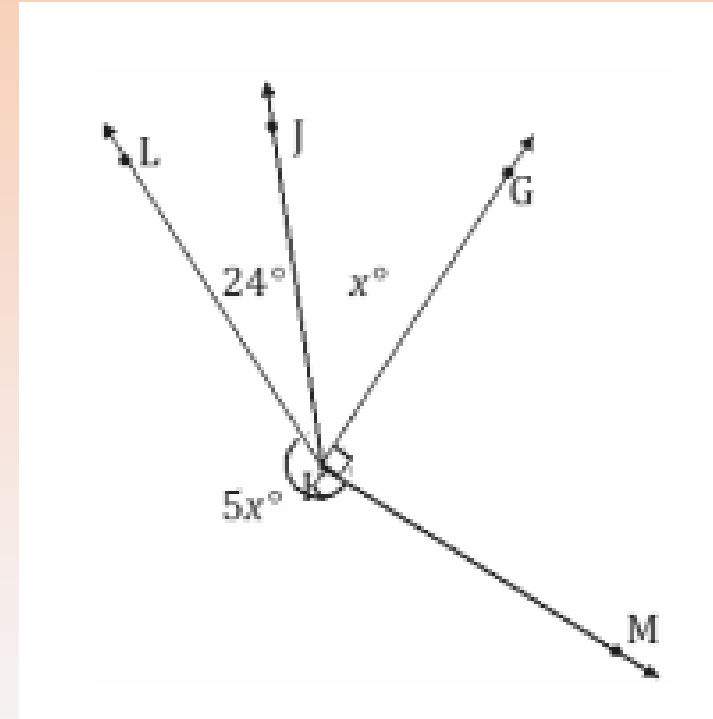


$$m\angle JAH = 2(9^\circ) = 18^\circ$$

$$m\angle GAF = 3(9^\circ) = 27^\circ$$

Exercise 3: (s.76)

In a complete sentence, describe the angle relationships in the diagram. Write an equation for the angle relationship shown in the figure and solve for x .



The sum of the degree measurements of $\angle LKJ$, $\angle JKG$, $\angle GKM$ and the arc that subtends $\angle LKM$ is 360° . Find the measure of $\angle JKG$.

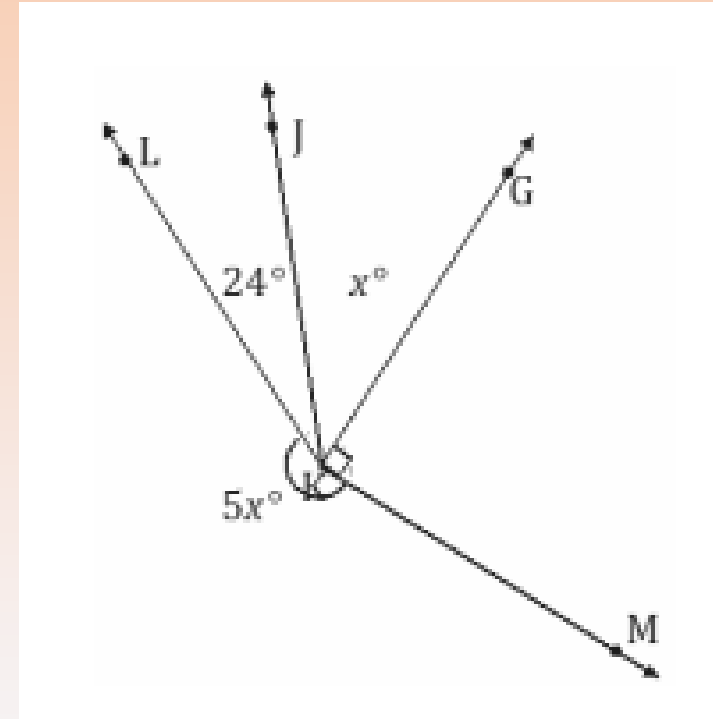
$$5x + 24 + x + 90 = 360$$

$$6x + 114 = 360$$

$$6x + 114 - 114 = 360 - 114$$

$$6x = 246$$

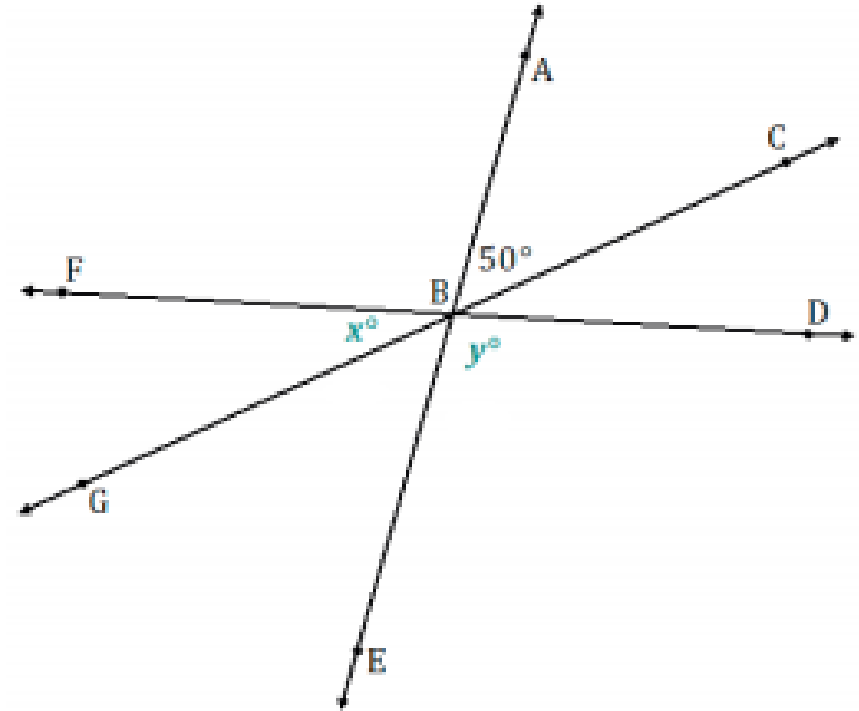
$$\left(\frac{1}{6}\right) 6x = \left(\frac{1}{6}\right) 246 \quad x = 41^\circ$$



Example 4: (s.77)

In the accompanying diagram, the measure of $\angle DBE$ is four times the measure of $\angle FBG$.

a. Label $\angle DBE$ as y° and $\angle FBG$ as x° . Write an equation that describes the relationship between $\angle DBE$ and $\angle FBG$.



$$y = 4x$$

b. Find the value of x .

$$50 + x + 4x = 180$$

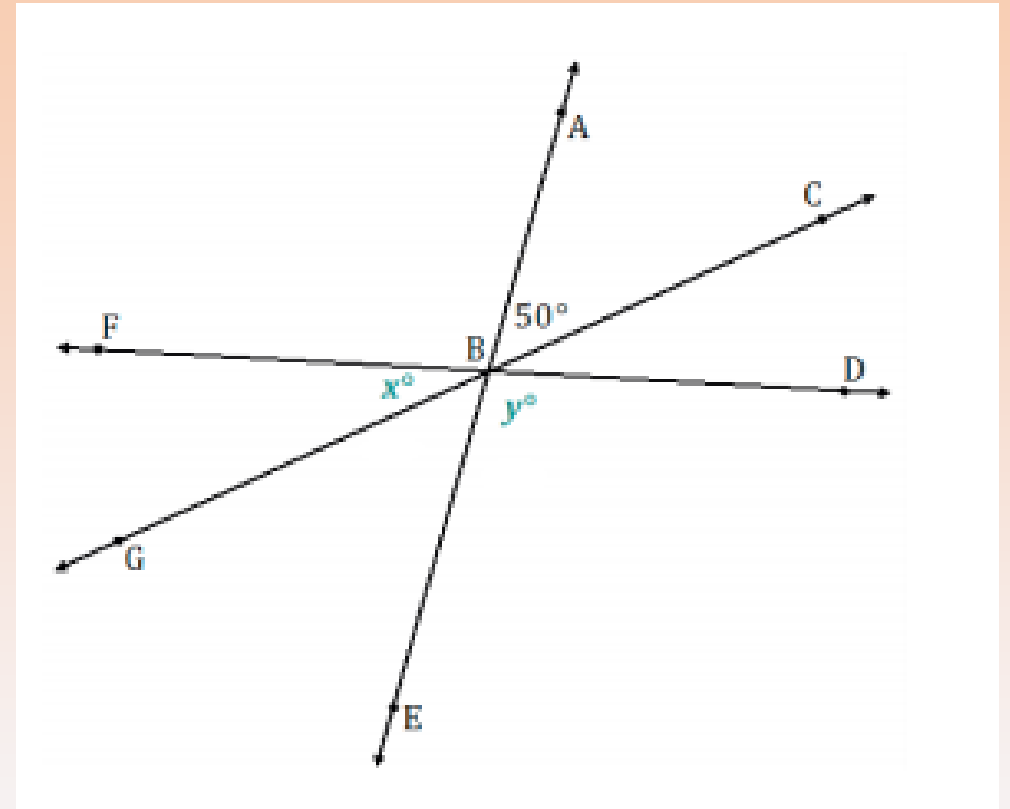
$$50 + 5x = 180$$

$$5x + 50 - 50 = 180 - 50$$

$$5x = 130$$

$$\left(\frac{1}{5}\right) 5x = \left(\frac{1}{5}\right) 130$$

$$x = 26$$



c. Find the measures of $\angle FBG$, $\angle CBD$, $\angle ABF$, $\angle GBE$, and $\angle DBE$.

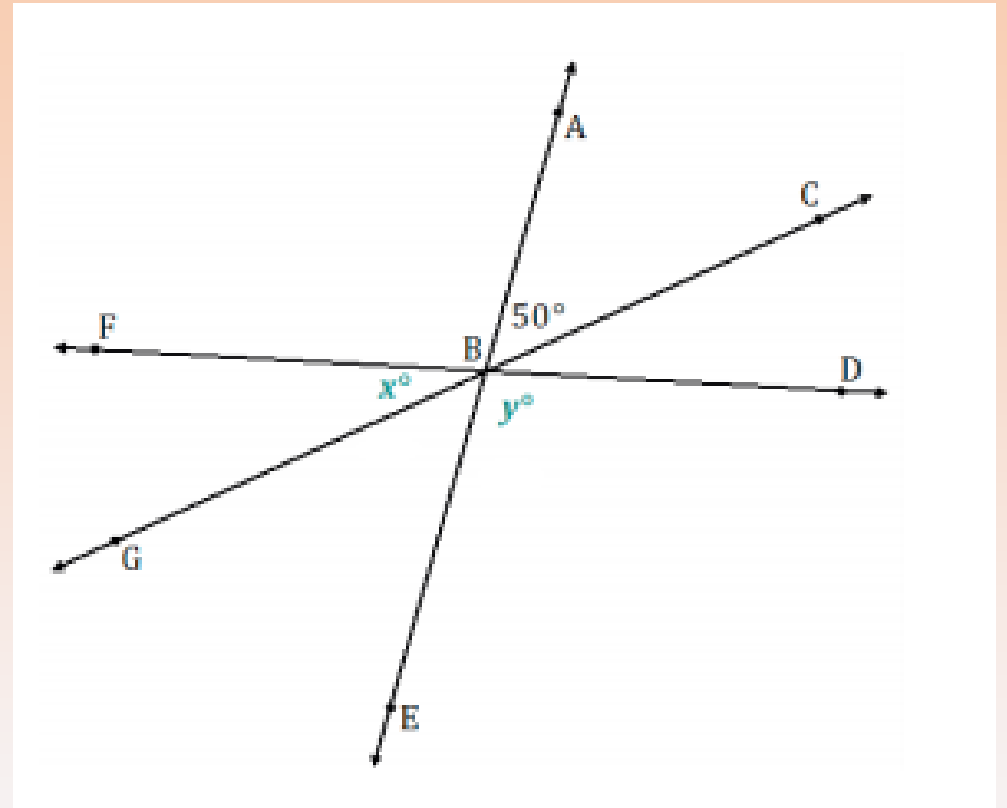
$$m\angle FBG = 26^\circ$$

$$m\angle CBD = 26^\circ$$

$$m\angle ABF = 4(26^\circ) = 104^\circ$$

$$m\angle GBE = 50^\circ$$

$$m\angle DBE = 104^\circ$$



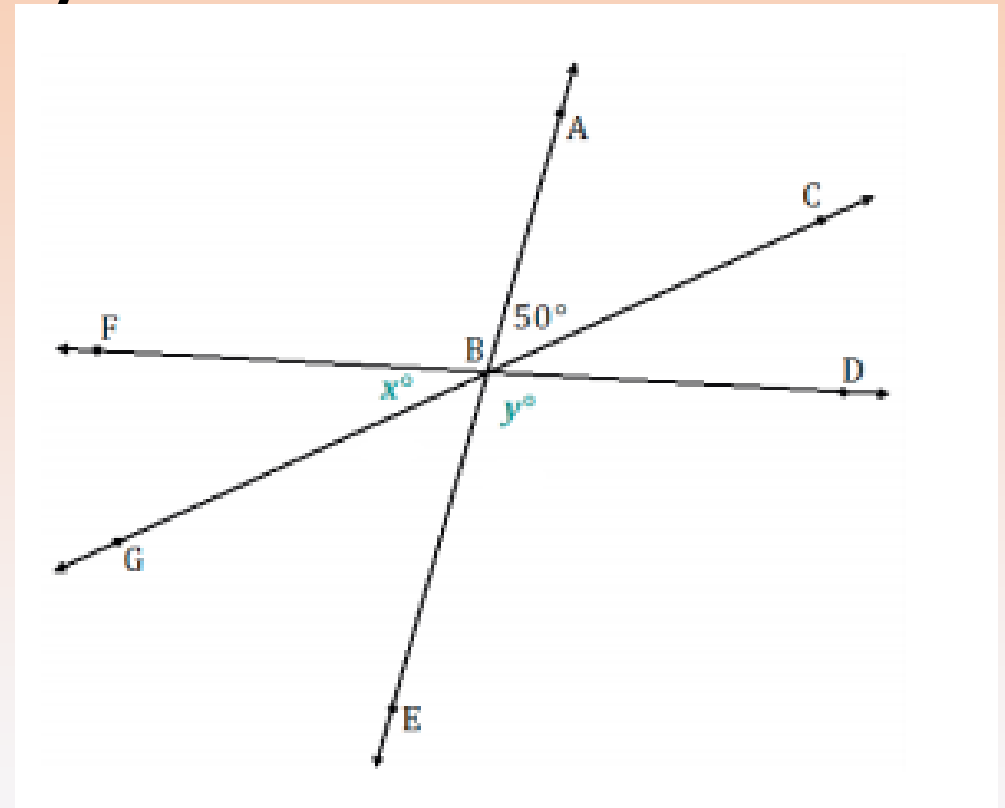
d. What is the measure of $\angle ABG$? Identify the angle relationship used to get your answer.

$$\angle ABG = \angle ABF + \angle FBG$$

$$\angle ABG = 104 + 26$$

$$\angle ABG = 130$$

$$m\angle ABG = 130^\circ$$



To determine the measure of $\angle ABG$, you need to add the measures of adjacent angles $\angle ABF$ and $\angle FBG$.

Problem Set:

(s.78 -80)