Lesson 15: Graphing Solutions to Inequalities

Student Outcomes:

- Students graph solutions to inequalities taking care to interpret the solutions in the context of the problem.

Bell Work: Games at the carnival cost $3 each. The prizes awarded to winners cost $145.65. How many games must be played to make at least $50?

$$3g - 145.65 \geq 50$$

$$3g - 145.65 + 145.65 \geq 50 + 145.65$$

$$3g + 0 \geq 195.65$$

$$\left(\frac{1}{3}\right)(3g) \geq \left(\frac{1}{3}\right)(195.65)$$

$$g \geq 65.217$$

Notes:

An inequality is a mathematical sentence that contains the symbols <, >, =, ≤, or ≥.

Inequalities
< is less than
> Is greater than
≤ is less than or equal to; is at most
≥ is greater than or equal to; is at least

Notes:

Inequalities with variables can be graphed. The graph give you a quick picture of all the mystery numbers on the number line that will work. The following are steps for graphing an inequality on a number line.

Notes:

1. Locate the number of the inequality on the number line.

2. If the inequality is either > or <, circle the number. If the inequality is either ≥ or ≤, circle and shade the number.

3. If the inequality is either > or ≥, shade the number line to the right of the number. If the inequality is either < or ≤, shade the number line to the left of the number.

Rapid White Board Exchange (10 minutes): Inequalities Students complete a rapid whiteboard exchange where they practice their knowledge of solving linear inequalities in the form $px + q > r$ and $p(x + q) > r$.

Determine the value(s) of the variable. Set 1

1. $x + 1 > 8$ $x > 7$
2. $x + 3 > 8$ $x > 5$
3. $x + 10 > 8$ $x > -2$
4. $x - 2 > 3$ $x > 5$
5. $x - 4 > 3$ $x > 7$

Determine the value(s) of the variable. Set 2

1. $3x \leq 15$ $x \leq 5$
2. $3x \leq 21$ $x \leq 7$
3. $-x \leq 4$ $x \geq -4$
4. $-2x \leq 4$ $x \geq -2$
5. $-x \leq -4$ $x \geq 4$

Determine the value(s) of the variable. Set 3

1. $\frac{1}{2}x < 1$ x < 2
2. $\frac{1}{2}x < 3$ x < 6
3. $-\frac{1}{5}x < 2$ x > -10
4. $-\frac{2}{5}x < 2$ x > -5
5. $-\frac{3}{5}x < 3$ x > -5

Determine the value(s) of the variable. Set 4

1. $2x + 4 \geq 8$ $x \geq 2$
2. $2x - 3 \geq 5$ $x \geq 4$
3. $-2x + 1 \geq 7$ $x \leq -3$
4. $-3x + 1 \geq -8$ $x \leq 3$
5. $-3x - 5 \geq 10$ $x \leq -5$

Determine the value(s) of the variable. Set 5

1. $2x - 0.5 > 5.5$ $x > 3$
2. $3x + 1.5 > 4.5$ $x > 2$
3. $5x - 3 > 4.5$ $x > 1.5$
4. $-5x + 2 > 8.5$ $x < -1.3$
5. $-9x - 3.5 > 1$ $x < -0.5$

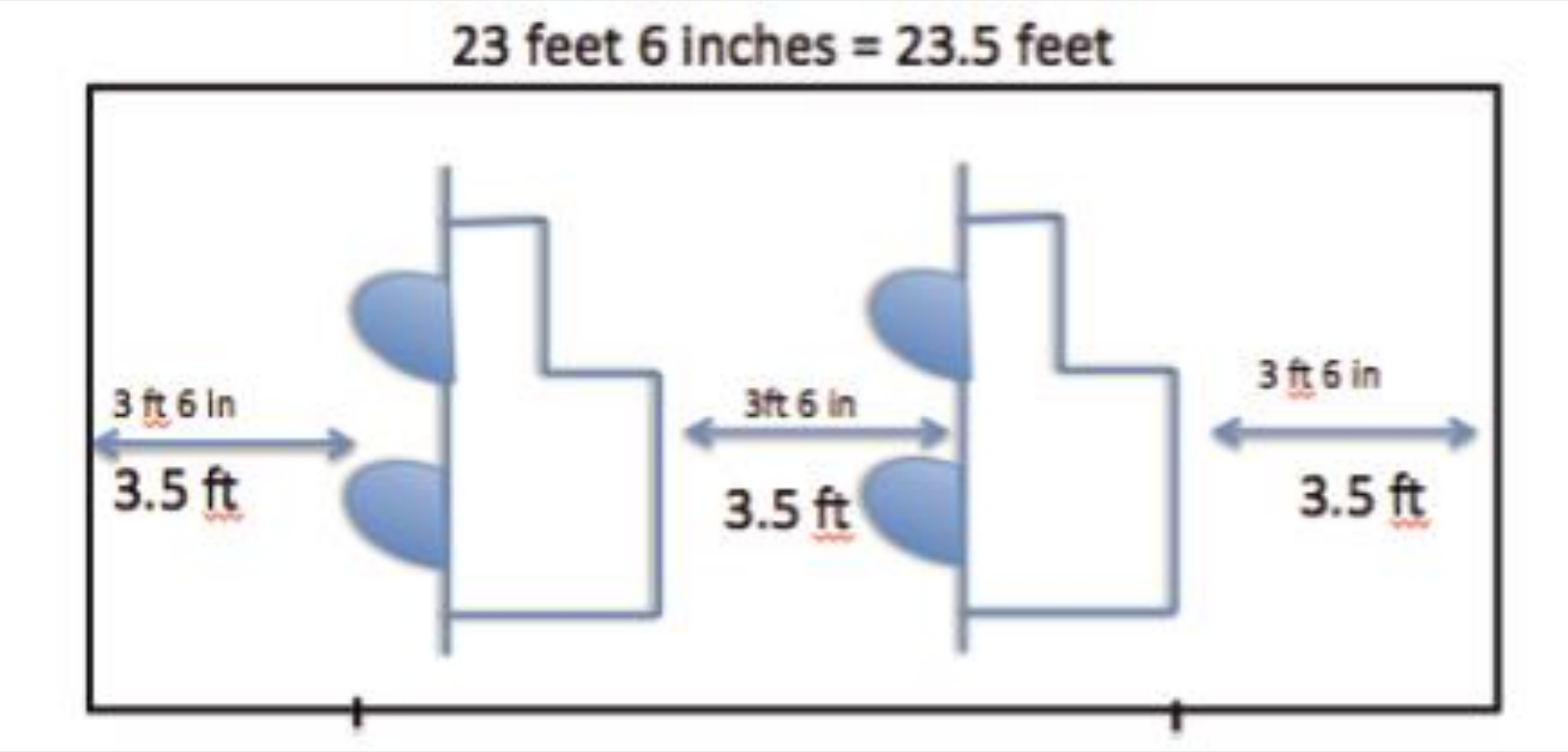
Determine the value(s) of the variable. Set 6

1. $2(x + 3) \leq 4$ $x \leq -1$
2. $3(x + 3) \leq 6$ $x \leq -1$
3. $4(x + 3) \leq 8$ $x \leq -1$
4. $-5(x - 3) \leq -10$ $x \geq 5$
5. $-2(x + 3) \leq 8$ $x \geq -7$

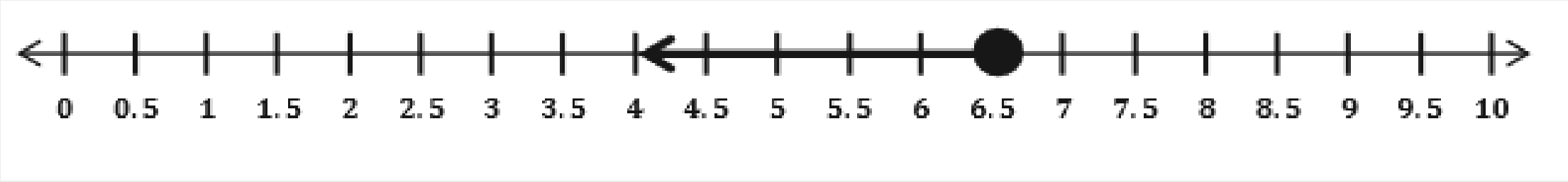
Exercise 1: (s.97)

1. Two identical cars need to fit into a small garage. The opening is 23 feet 6 inches wide, and there must be at least 3 feet 6 inches of clearance between the cars and between the edges of the garage. How wide can the cars be?

Draw a diagram to illustrate the problem.



Find all the widths that the cars could be.



Example: (s.97-98)

A local car dealership is trying to sell all of the cars that are on the lot. Currently, there are 525 cars on the lot, and the general manager estimates that they will consistently sell 50 cars per week. Estimate how many weeks it will take for the number of cars on the lot to be less than 75.

Write an inequality that can be used to find the number of full weeks, w, it will take for the number of cars to be less than 75. Since w is the number of full or complete weeks, w = 1 means at the end of week 1.

$$525 - 50w < 75$$

Solve and graph the inequality.

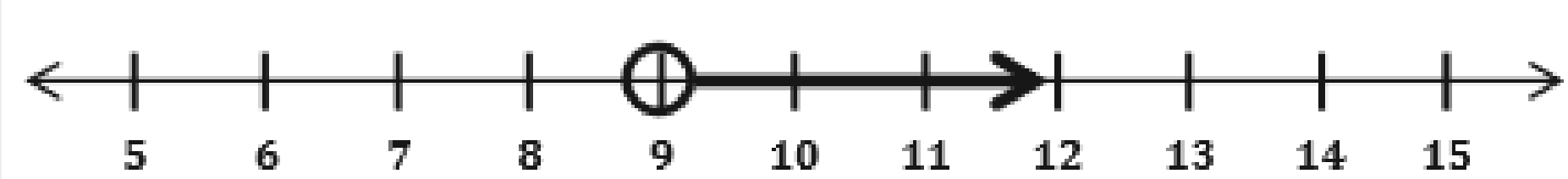
$$525 - 50w < 75$$

$$-50w + 525 - 525 < 75 - 525$$

$$-50w + 0 < -450$$

$$\left(-\frac{1}{50}\right)(50w) > \left(-\frac{1}{50}\right)(-450)$$

$$W > 9$$



Interpret the solution in the context of the problem.

The dealership can sell 50 cars per week for more than 9 weeks to have less than 75 cars remaining on the lot.

Verify the solution:

w = 9

$525 - 50w < 75$

$525 - 50(9) < 75$

$525 - 450 < 75$

$75 < 75$

False

w = 10

$525 - 50w < 75$

$525 - 5(10) < 75$

$525 - 500 < 75$

$25 < 75$

True

Exercise 2: (s.98)

2. The cost of renting a car is $25 per day plus a one-time fee of $75.50 for insurance. How many days can the car be rented if the total cost is to be no more than $525?

a. Write an inequality to model the situation.

Let x represent the number of days the car is rented.

$$25x + 75.50 \leq 525$$

b. Solve and graph the inequality.

$$25x + 75.50 \leq 525$$

$$25x + 75.50 - 75.50 \leq 525 - 75.50$$

$$25x + 0 \leq 449.50$$

$$\left(\frac{1}{25}\right)(25x) \leq \left(\frac{1}{25}\right)(449.50)$$

$$x \leq 17.98$$

c. Interpret the solution in the context of the problem.

The car can be rented for 17 days or fewer and stay within the amount of $525. The number of days is an integer. The 18th day would put the cost over $525, and since the fee is charged per day, the solution set includes whole numbers.

3. Mrs. Smith decides to buy three sweaters and a pair of jeans. She has $120 in her wallet. If the price of the jeans is $35, what is the highest possible price of a sweater, if each sweater is the same price?

Let w represent the price of one sweater.

$$3w + 35 \leq 120$$

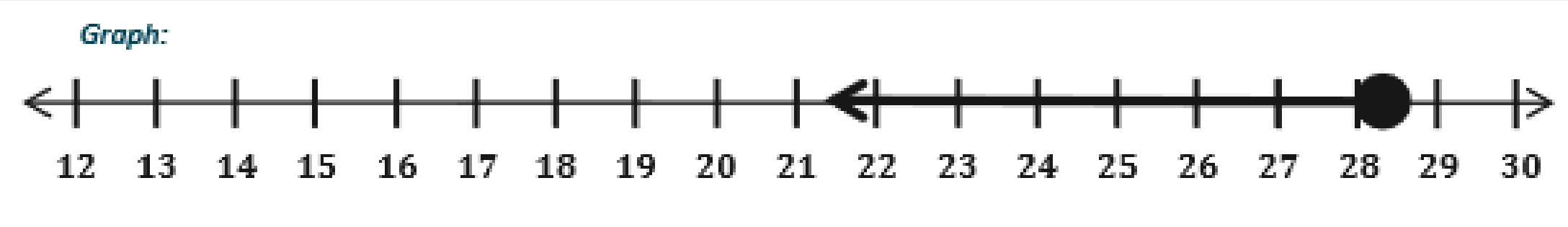
$$3w + 35 - 35 \leq 120 - 35$$

$$3w + 0 \leq 85$$

$$\left(\frac{1}{3}\right)(3w) \leq \left(\frac{1}{3}\right)(85)$$

$$w \leq 28.33$$

Draw a graph of the solution and explain what it means.



Solution: The highest price Mrs. Smith can pay for a sweater and have enough money is $28.33.

4. The members of the Select Chorus agree to buy at least 250 tickets for an outside concert. They buy 20 fewer lawn tickets than balcony tickets. What is the least number of balcony tickets bought?

Let b represent the number of balcony tickets.

Then b – 20 represents the number of lawn tickets.

$$b + b - 20 \geq 250$$

$$2b - 20 \geq 250$$

$$2b - 20 + 20 \geq 250 + 20$$

$$2b + 0 \geq 270$$

$$\left(\frac{1}{2}\right)(2b) \geq \left(\frac{1}{2}\right)(270)$$

$$b \geq 135$$

Draw a graph of the solution and explain what it means.

Graph:

Solution: The least number of balcony tickets bought is 135. The answers need to be integers.

5. Samuel needs $29 to download some songs and movies on his MP3 player. His mother agrees to pay him $6 an hour for raking leaves in addition to his $5 weekly allowance. What is the minimum number of hours Samuel must work in one week to have enough money to purchase the songs and movies?

Let h represent the number of hours Samuel rakes leaves.

$$6h + 5 \geq 29$$

$$6h + 5 - 5 \geq 29 - 5$$

$$6h + 0 \geq 24$$

$$\left(\frac{1}{6}\right)(6h) \geq \left(\frac{1}{6}\right)(24)$$

$$h \geq 4$$

Draw a graph of the solution and explain what it means.

Graph:

Solution: Samuel needs to rake leaves at least 4 hours to earn $29. Any amount of time over 4 hours will earn him extra money.

Closing:

Why do we use rays when graphing the solutions of an inequality on a number line?

When graphing the solution of an inequality on a number line, how do you determine what type of circle to use (open or closed)?

When graphing the solution of an inequality on a number line, how do you determine the direction of the arrow?

Problem Set:
(s.101)