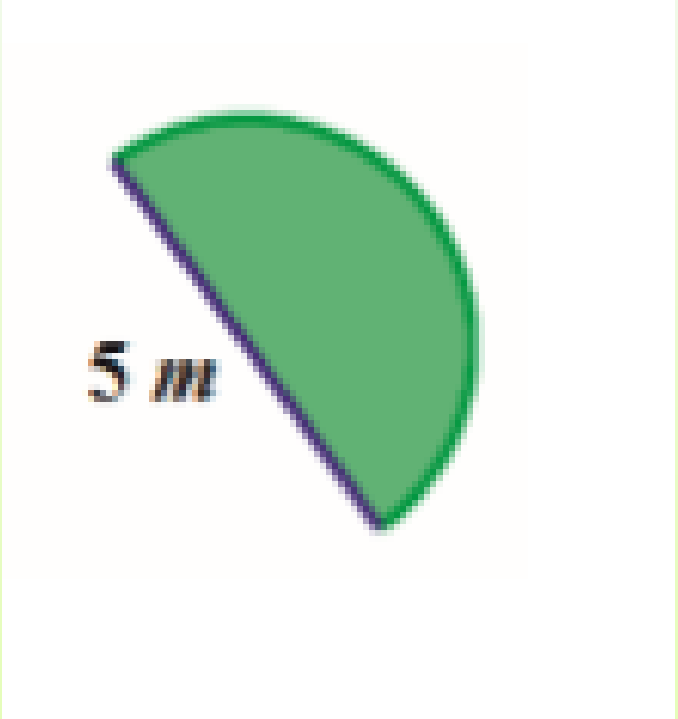
Lesson 19: Unknown Area Problems on the Coordinate Plane

Student Outcomes:

- Students find the areas of triangles and simple polygonal regions in the coordinate plane with vertices at grid points by composing into rectangles and decomposing into triangles and quadrilaterals.

Bell Work:

Ken's landscape gardening business creates odd-shaped lawns that include semicircles. Find the area of this semicircular section of the lawn in this design. Use $\frac{22}{7}$ for π.



If the diameter is 5 m, then the radius is $\frac{5}{2}\ m$.

$$A = \frac{1}{2}\pi r^2 \approx \frac{1}{2} \bullet \frac{22}{7} \bullet \left(\frac{5}{2}\ m\right)^2$$

$$A \approx \frac{1}{2} \bullet \frac{22}{7} \bullet \frac{25}{4} m^2 \approx \frac{550}{56} m2 \approx 9.8\ m^2$$

Notes:

Area of a triangle:

$A = \frac{1}{2}(bh)$

Area of rectangle:

$A = lw$

Area of a parallelogram:

$A = bh$

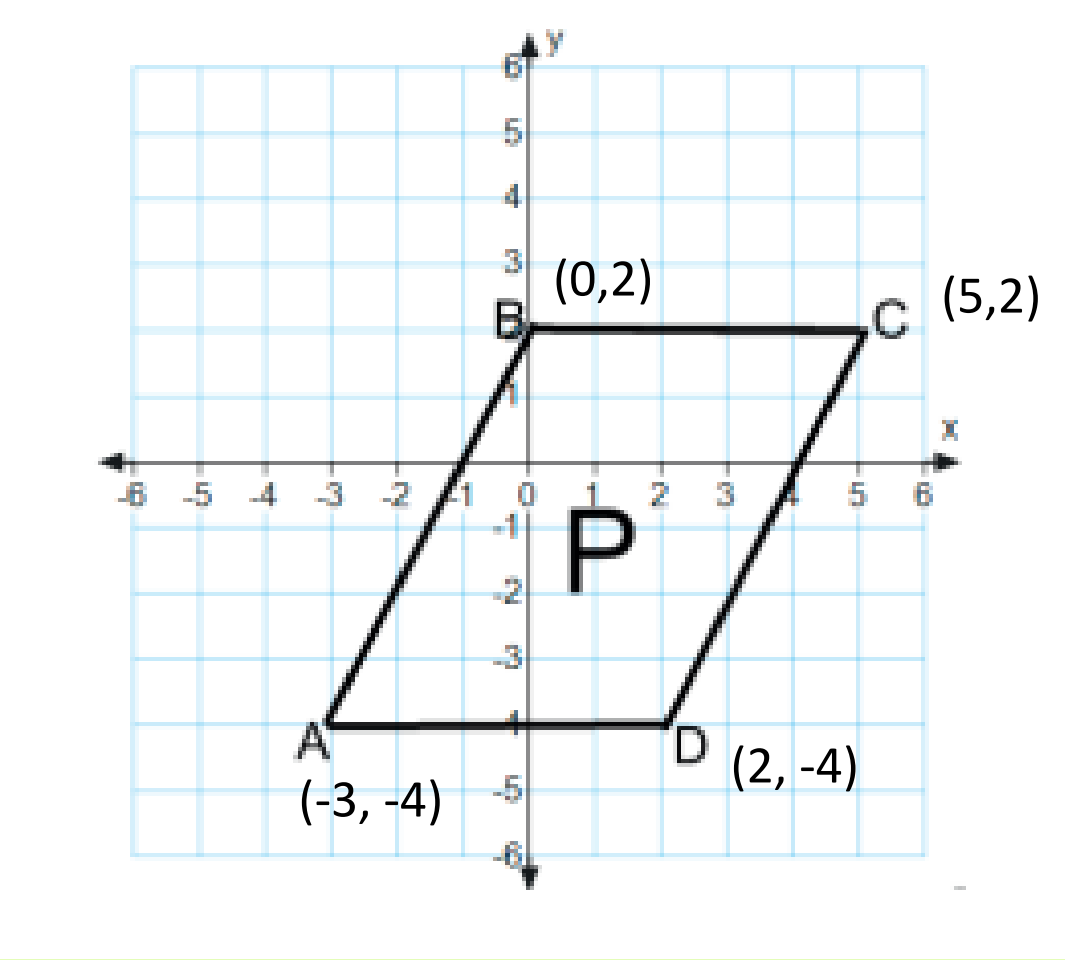
Area of a trapezoid:

$\frac{1}{2} \times (b_1 + b_2) \times \text{height}$

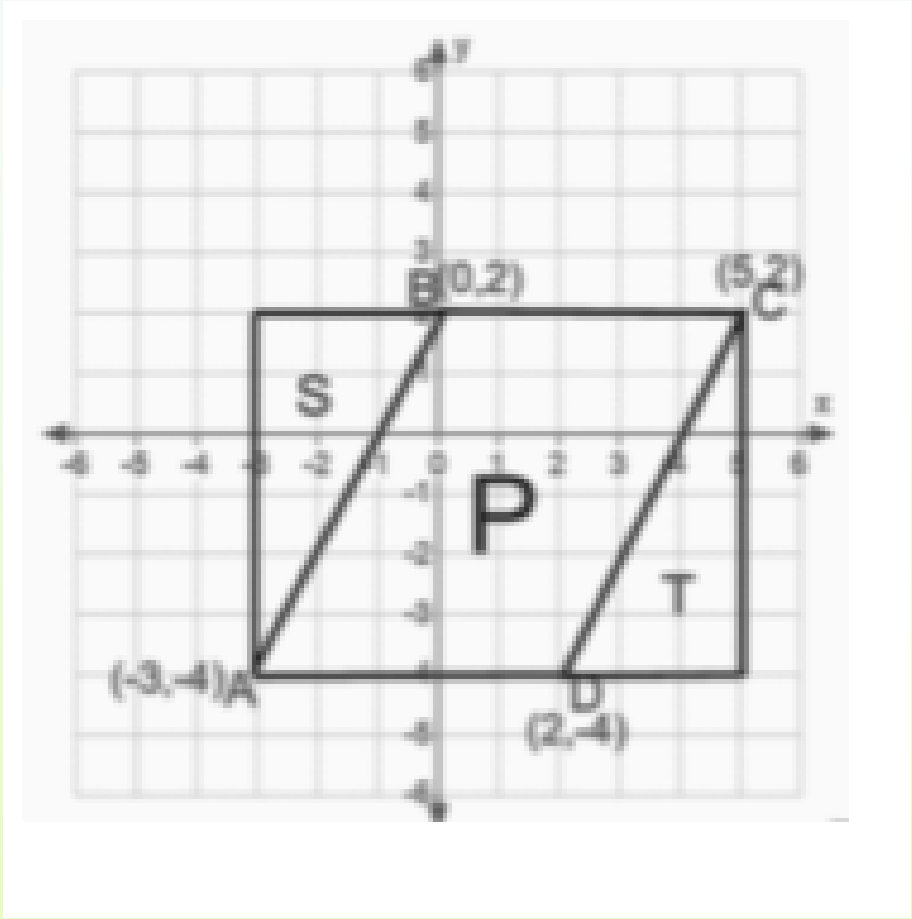
Example: Area of a Parallelogram (s.127 - 128)

The coordinate plane below contains figure P, parallelogram ABCD.

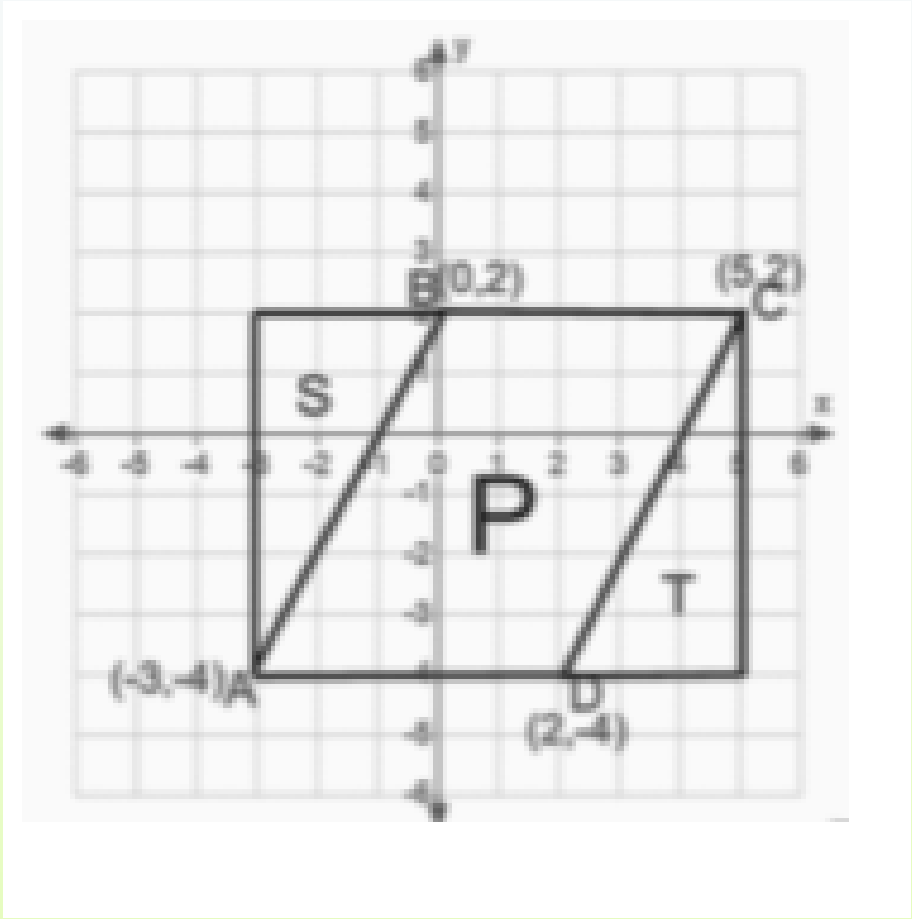
a. Write the ordered pairs of each of the vertices next to the vertex points.



b. Draw a rectangle surrounding figure P that has vertex points of A and C. Label the two triangles in the figure as S and T.



c. Find the area of the rectangle.



Base = 8 units

Height = 6 units

Area = 8 units × 6 units = 48 sq. units

d. Find the area of each triangle.

Figure S

Base = 3 units

Height = 6 units

Area = $\frac{1}{2}\ x\ 3\ units\ x\ 6\ units$

= 9 sq. units

Figure T

Base = 3 units

Height = 6 units

Area = $\frac{1}{2}\ x\ 3\ units\ x\ 6\ units$

= 9 sq. units

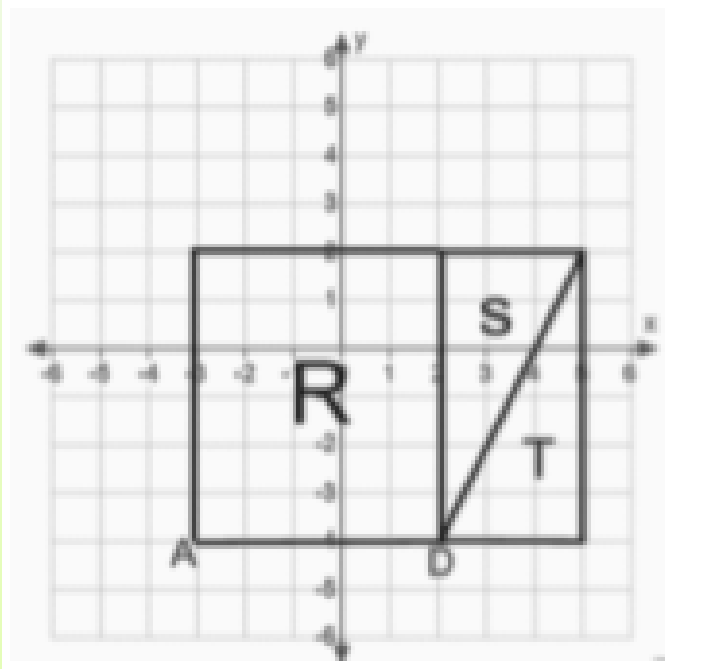
e. Use these areas to find the area of parallelogram ABCD.

Area P = Area of rectangle – Area S – Area T

= 48 sq. units – 9 sq. units – 9 sq. units = 30 sq. units

The coordinate plane below contains figure R, a rectangle with the same base as the parallelogram above.

f. Draw triangles S and T and connect to figure R so that you create a rectangle that is the same size as the rectangle you created on the first coordinate plane.

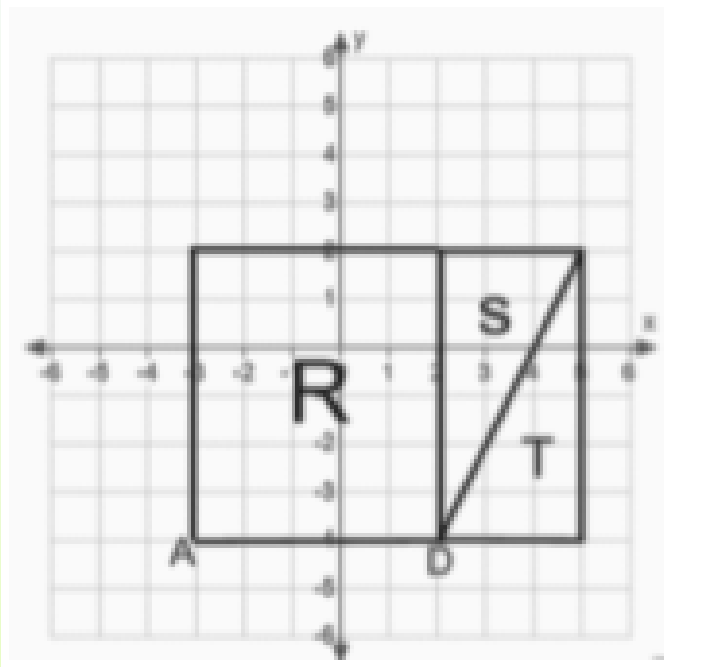


g. Find the area of rectangle R.

Base = 5 units

Height = 6 units

Area = 30 sq. units

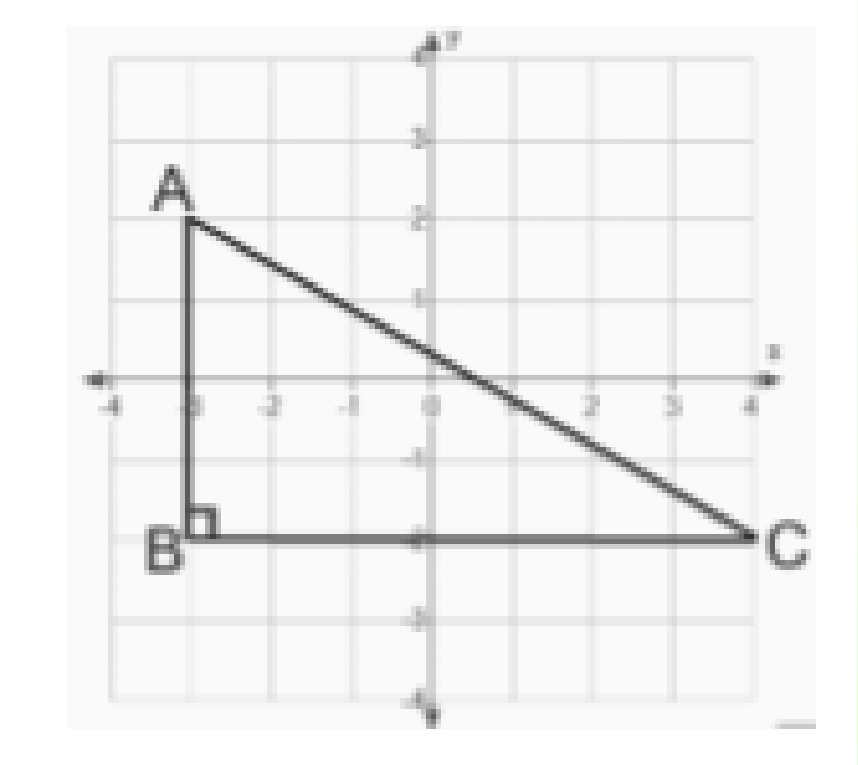


h. What do figures R and P have in common?

They have the same area.
They share the same base
and have the same height.

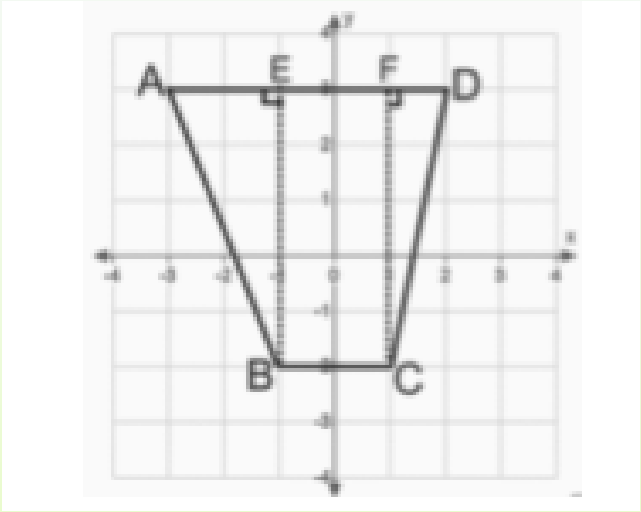
Exercises: (s.129)

1. Find the area of triangle ABC.



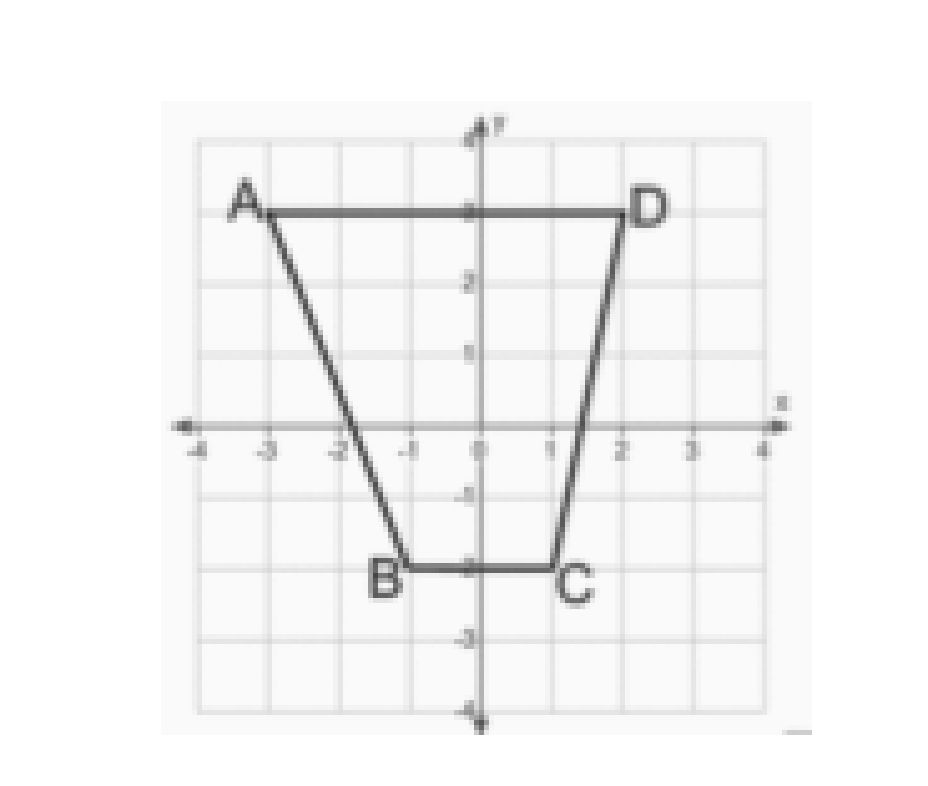
$$A = \frac{1}{2} \bullet 7\ units \bullet 4\ units\ = 14\ sq\ units$$

2. Find the area of quadrilateral ABCD two different ways.



$$\frac{1}{2} \bullet 2 \bullet 5 + 2 \bullet 5 + \frac{1}{2} \bullet 1 \bullet 5 = 5 + 10 + 2.5 = 17.5$$

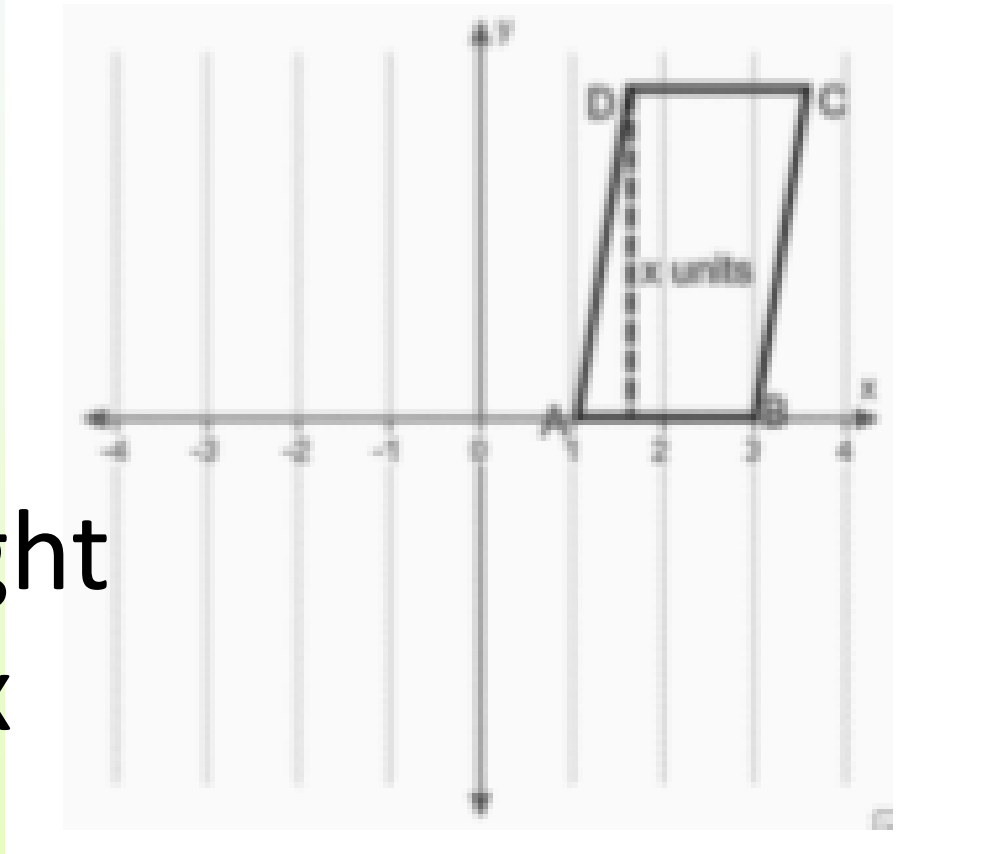
The area is 17.5 sq. units



$$\frac{1}{2} \bullet (5 + 2) \bullet 5 = 17.5$$

The area is 17.5 sq. units

3. The area of quadrilateral ABCD is 12 sq. units. Find x.

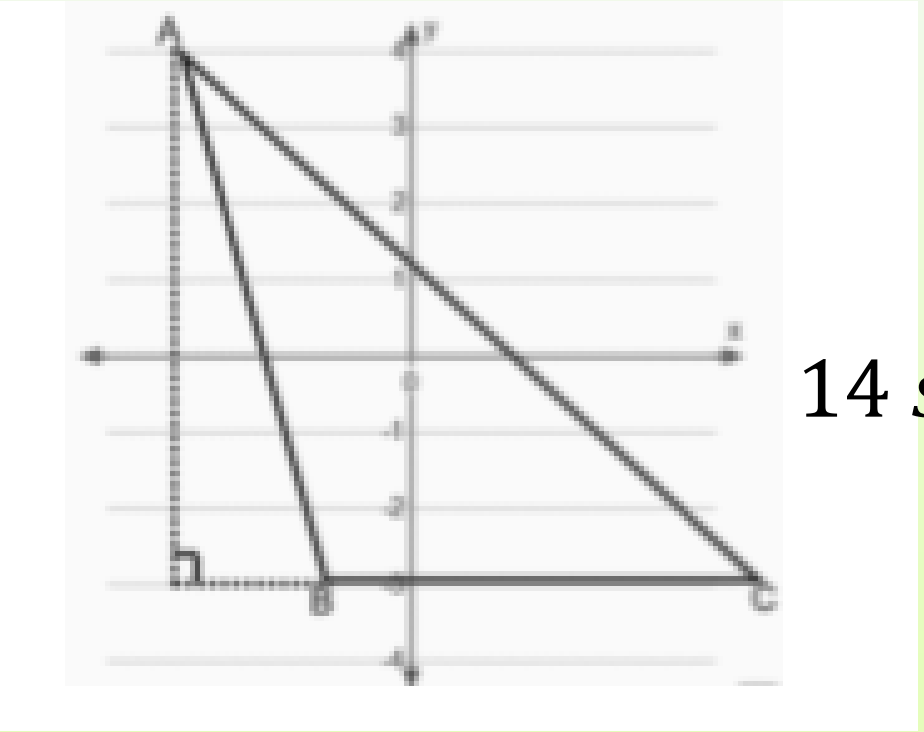


Area = base x height

12 sq. units = 2x

6 units = x

4. The area of triangle ABC is 14 sq. units. Find the length of side $\overline{BC}$.

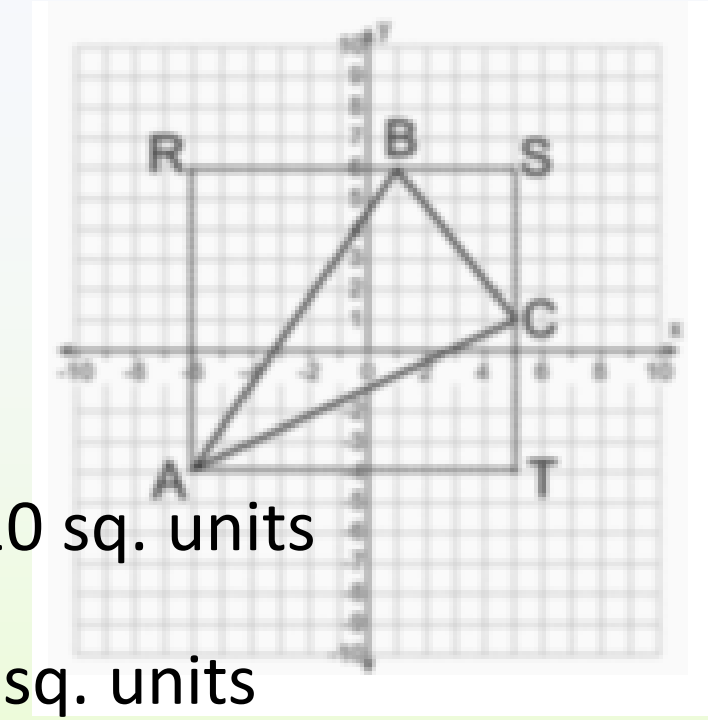


$$Area = \frac{1}{2} x\ base\ x\ height$$

$$14\ sq.units = \frac{1}{2} x\ BC\ x\ (7\ units)$$

$$BC = 4\ units$$

5. Find the area of triangle ABC.



Area of rectangle ARST = 11 units x 10 units = 110 sq. units

Area of triangle ARB = $\frac{1}{2}$ x 7 units x 10 units = 35 sq. units

Area of triangle BSC = $\frac{1}{2}$ x 4 units x 5 units = 10 sq. units

Area of triangle ATC = $\frac{1}{2}$ x 11 units x 5 units = 27.5 units

Area of triangle ABC = Area of ARST – Area of ARB – Area of BSC – Area of ATC = 37.5 sq. units

Closing: Vocabulary:

Quadrilateral

Rectangle

Semicircle

Parallelogram

Square

Diameter of a Circle

Trapezoid

Altitude and base of a triangle

Area Formulas:

Area of a parallelogram = base x height

Area of a triangle = ½ x base x height

Area of a circle = π x r^2

Area of a rectangle = base x height

Area of a trapezoid = ½ x (base 1 + base 2) x height

- Why is it useful to have a figure on a coordinate plane?

  The scale can be used to measure the base and height.

- What are some methods for finding the area of a quadrilateral?

  Use a known area formula, deconstruct the figure into shapes with known area formulas, make the figure a part of a larger shape and then subtract areas.

Problem Set: (s.131- 133)