Lesson 18: More Problems on Area and Circumference

Student Outcomes:

- Students examine the meaning of quarter circle and semicircle.
- Students solve area and perimeter problems for regions made out of rectangles, quarter circles, semicircles, and circles, including solving for unknown lengths when the area or perimeter is given.

Bell Work:

A circle has a diameter of 30 inches. Find the area of the circle using the approximation $\pi = 3.14$.

$$A = \pi r^2$$

706.5 inches^2

Notes:

Circumference of a circle:

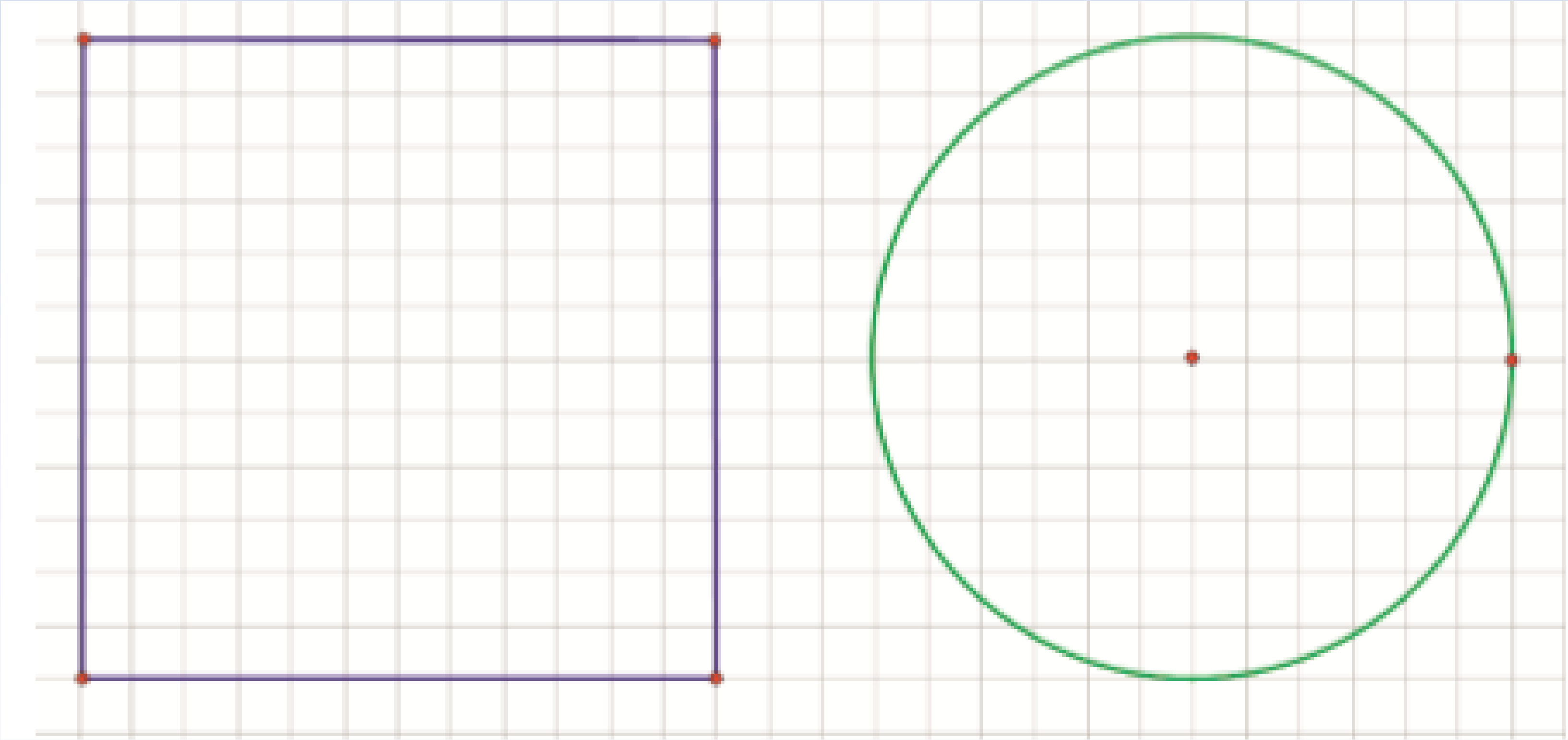
$$C = 2\pi r$$

or

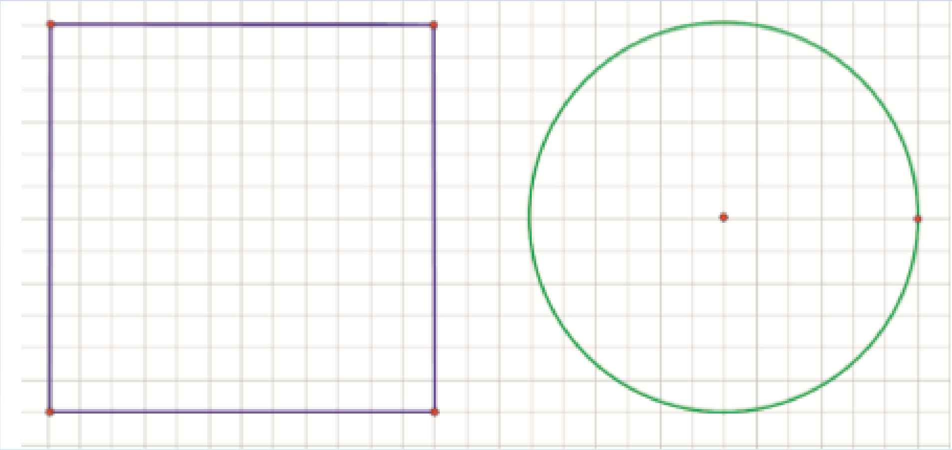
$$C = \pi d$$

Opening Exercise (S.119)

Draw a circle with a diameter of 12 cm and a square with a side length of 12 cm on grid paper.



Determine the area of the square and the circle.



Area of Square: $A = (12\ cm)^2 = 144\ cm^2$

Area of Circle: $A = \pi \bullet (6cm)^2 = 36\pi\ cm^2$

Brainstorm some methods for finding half the area of the square and half the area of the circle.

Some methods include folding in half and counting the grid squares and cutting each in half and counting the squares.

Find the area of half of the square and half of the circle, and explain to a partner how you arrived at the area.

The area of half of the square is 72 cm^2. The area of half of the circle is $18\pi\ cm^2$.

What is the ratio of the new area to the original area for the square and for the circle?

The ratio of the areas of the rectangle (half of the square) to the square is 72:144 or 1:2 . The ratio for the areas of the circles is $18\pi : 36\pi$ or 1:2.

Find the area of one-fourth of the square and one-fourth of the circle, first by folding and then by another method. What is the ratio of the new area to the original area for the square and for the circle?

Folding the square in half and then in half again will result in one-fourth of the original square. The resulting shape is a square with a side length of 6 cm and an area of 36 cm^2. Repeating the same process for the circle will result in an area of $9\pi\ cm^2$. The ratio for the areas of the squares is 36: 144 or 1: 4. The ratio for the areas of the circles is $9\pi : 36\pi$ or 1:4.

Write an algebraic expression that expresses the area of a semicircle and the area of a quarter circle.

Semicircle: $A = \frac{1}{2}\pi r^2$

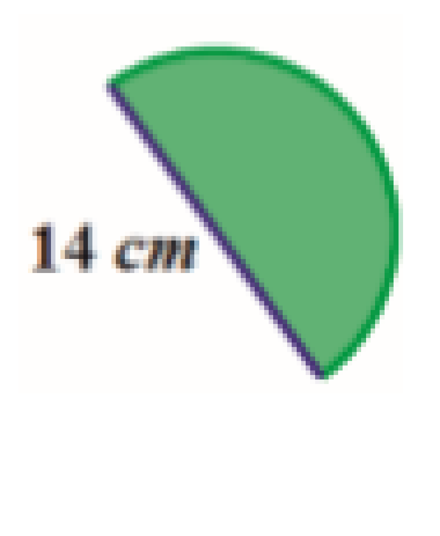
Quarter Circle: $A = \frac{1}{4}\pi r^2$

Example 1: (s.120)

Find the area of the following semicircle.

Use $\pi \approx \frac{22}{7}$.

If the diameter of the circle is 14 cm, then the radius is 7 cm. The area of the semicircle is half of the area of the circular region.

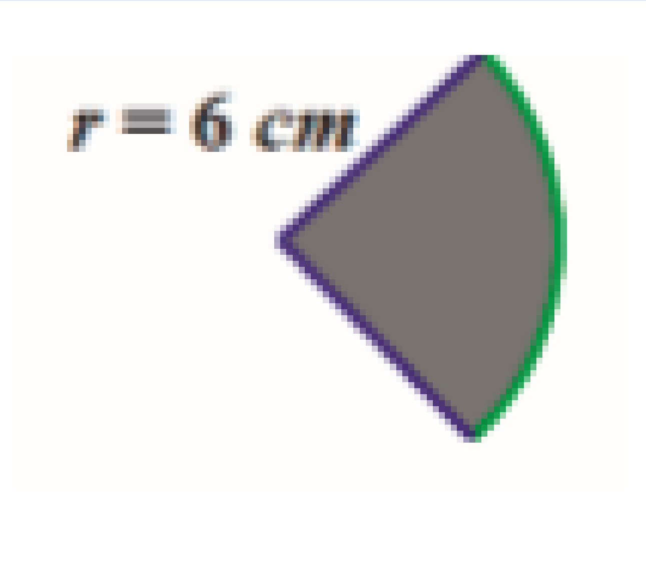


$$A \approx \frac{1}{2} \bullet \frac{22}{7} \bullet (7\ cm)^2$$

$$A \approx \frac{1}{2} \bullet \frac{22}{7} \bullet 49\ cm^2$$

$$A \approx 77\ cm^2$$

What is the area of the quarter circle? Use $\pi \approx \frac{22}{7}$.

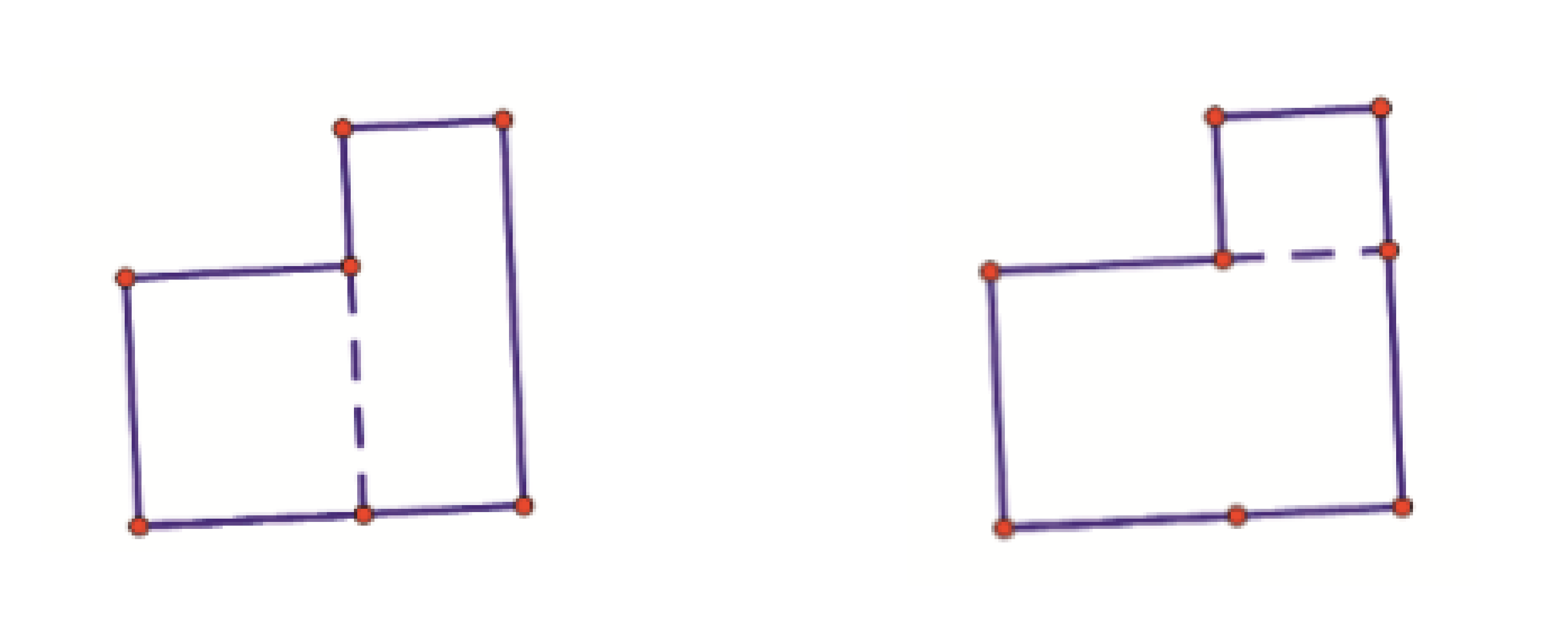


$$A \approx \frac{1}{4} \bullet \frac{22}{7} \bullet (6\,cm)^2$$

$$A \approx \frac{1}{4} \bullet \frac{22}{7} \bullet 36\,cm^2$$

$$A \approx \frac{198}{7} cm^2$$

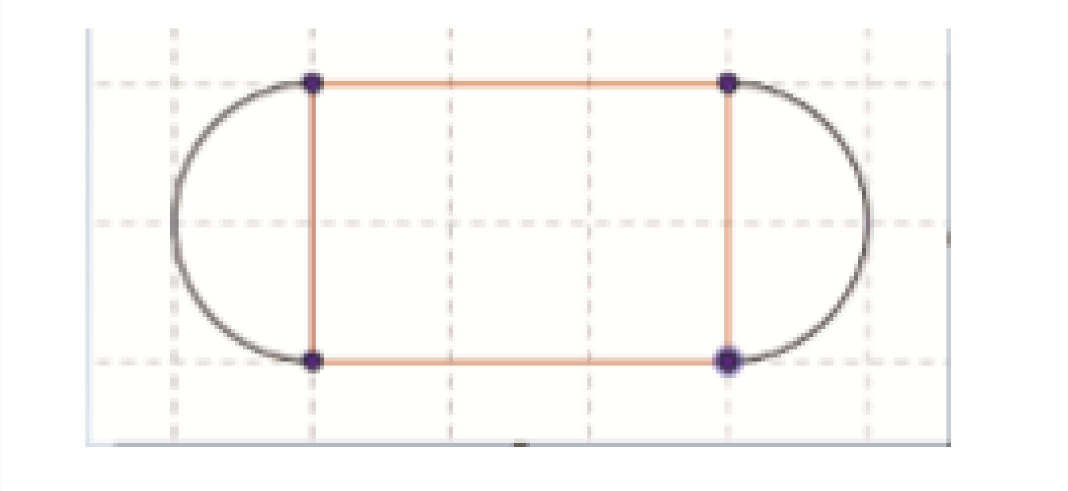
Composite Figures:

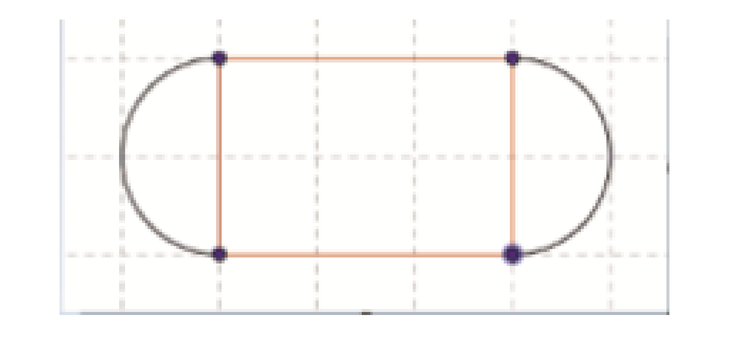


Example 2: (s 120 -121)

Marjorie is designing a new set of placemats for her dining room table. She sketched a drawing of the placement on graph paper. The diagram represents the area of the placemat consisting of a rectangle and two semicircles at either end. Each square on the grid measures 4 inches in length.

Find the area of the entire placemat. Explain your thinking regarding the solution to this problem.





The length of one side of the rectangular section is 12 inches in length, while the width is 8 inches. The radius of the semicircular region is 4 inches. The area of the rectangular part is (8 in)·(12 in)=96 in.^{2}. The total area must include the two semicircles on either end of the placemat. The area of the two semicircular regions is the same as the area of one circle with the same radius. The area of the circular region is $A = \pi \bullet (4\ in)^2$. In this problem, using $\pi \approx 3.14$ makes more sense because there are no fractions in the problem. The area of the semicircular regions is approximately 50.24 in^2. The total area for the placemat is the sum of the areas of the rectangular region and the two semicircular regions, which is approximately (96 + 50.24) in^2 = 146.24 in^2.

If Marjorie wants to make six placemats, how many square inches of fabric will she need? Assume there is no waste.

There are 6 placemats that are each 146.24 in^2, so the fabric needed for all is

$6 \bullet 146.24\ in^2 = 877.44\ in^2$.

Marjorie decides that she wants to sew on a contrasting band of material around the edge of the placemats. How much band material will Marjorie need?

The length of the band material needed will be the sum of the lengths of the two sides of the rectangular region and the circumference of the two semicircles (which is the same as the circumference of one circle with the same radius).

$$P = (l + l + 2\pi r)$$

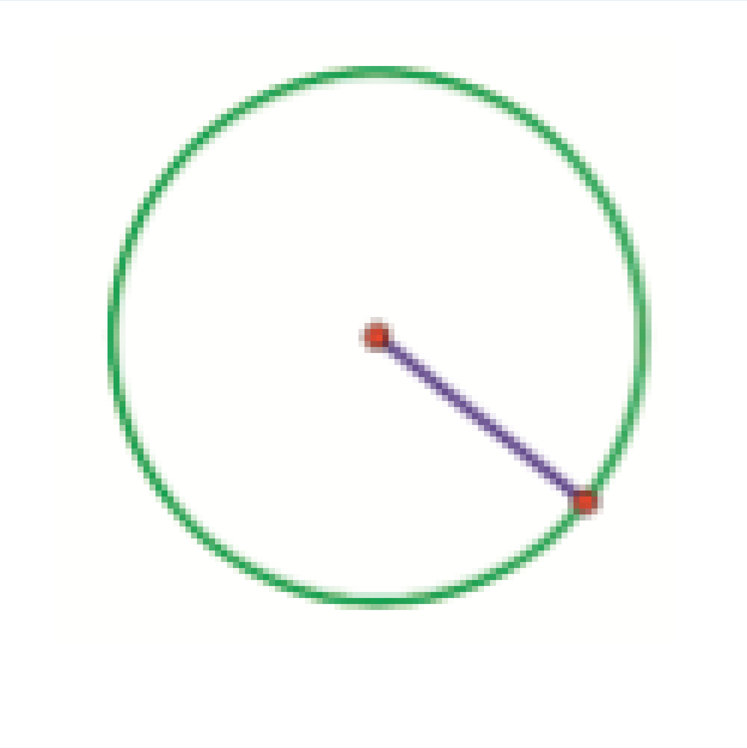
$$P = (12 + 12 + 2 \bullet \pi \bullet 4) = 49.12$$

The perimeter is 49.12 in^2.

Example 3: (s.121)

The circumference of a circle is $24\pi\ cm$. What is the exact area of the circle?

Draw a diagram to assist you in solving the problem.



What information is needed to solve the problem?

The radius is needed to find the area of the circle. Let the radius be r cm. Find the radius by using the circumference formula.

$$C = 2\pi r$$

$$24\pi = 2\pi r$$

$$\left(\frac{1}{2\pi}\right)24\pi = \left(\frac{1}{2\pi}\right)2\pi r$$

$$12 = r$$

The radius is 12 cm.

Next, find the area.

$$A = \pi r^2$$
$$A = \pi (12)^2$$
$$A = 144\pi$$

The exact area of the circle is 144π cm^2.

Exercises: (s.122)

1. Find the area of a circle with a diameter of 42 cm. Use $\pi \approx \frac{22}{7}$.

$$A = \pi r^2$$

$$A \approx \frac{22}{7}(21cm)^2$$

$$A \approx 1386\, cm^2$$

2. The circumference of a circle is 9π.

a. What is the diameter?

If $C = \pi d$, then $9\pi\ cm = \pi d$

Solving the equation for the diameter, d, $\frac{1}{\pi} \bullet 9\pi\ c. = \frac{1}{\pi}\pi \bullet d$

So, 9cm = d.

b. What is the radius?

If the diameter is 9 cm, then the radius is half of that or $\frac{9}{2}$ cm.

c. What is the area?

The area of the circle is

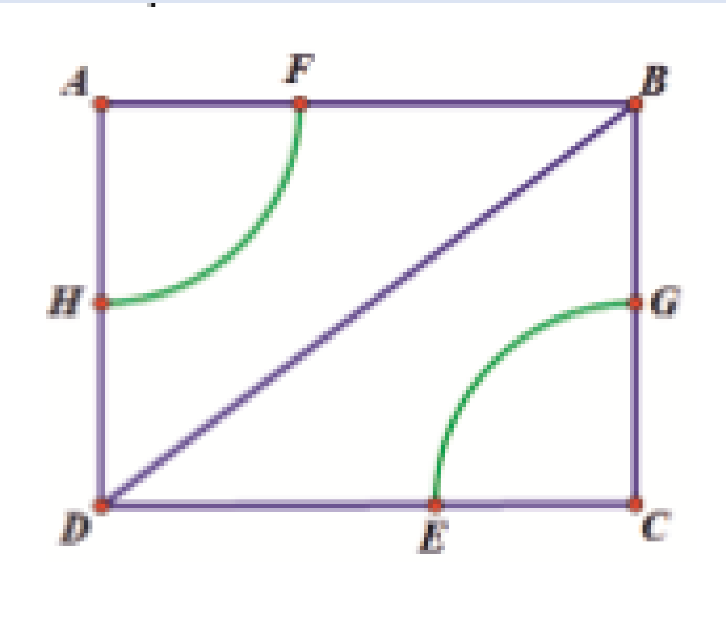
$$A = \pi \bullet \left(\frac{9}{2} cm\right)^2, so$$

$$A = \frac{81}{4}\pi\ cm^2.$$

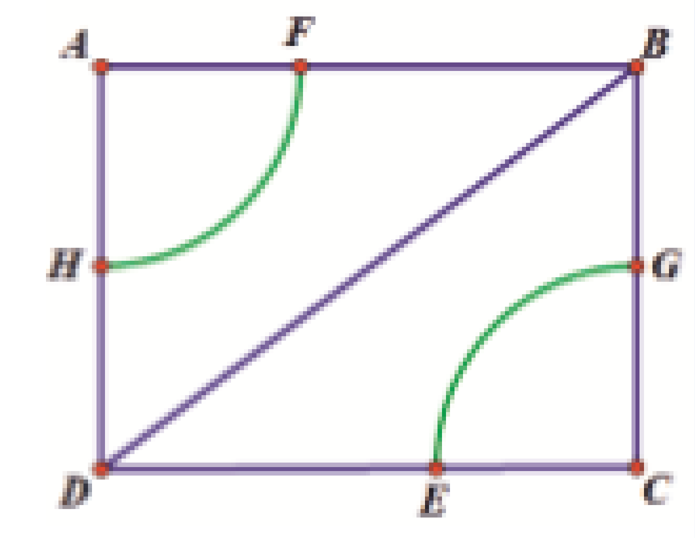
3. If students only know the radius of a circle, what other measures could they determine? Explain how students would use the radius to find the other parts.

If students know the radius, then they can find the diameter. The diameter is twice as long as the radius. The circumference can be found by doubling the radius and multiplying the result by π. The area can be found by multiplying the radius times itself and then multiplying that product by π.

4. Find the area in the rectangle between the two quarter circles if AF = 7 ft, FB = 9 ft, and HD = 7 ft. Use $\pi \approx \frac{22}{7}$. Each quarter circle in the top-left and lower-right corners have the same radius.



The area between the quarter circles can be found by subtracting the area of the two quarter circles from the area of the rectangle. The area of the rectangle is the product of the length and the width. Side AB has a length of 16 ft and Side AD has a length of 14 ft. The area of the rectangle is $A = 16\ ft \bullet 14\ ft = 224\ ft^2$. The area of the two quarter circles is the same as the area of a semicircle, which is half the area of a circle. $A = \frac{1}{2}\pi^2$.



$$A \approx \frac{1}{2} \bullet \frac{22}{7} \bullet (7\ ft)^2$$

$$A \approx \frac{1}{2} \bullet \frac{22}{7} \bullet 49\ ft^2$$

$$A \approx 77\ ft^2$$

The area between the two quarter circles is 224 ft^2 – 77 ft^2 = 147 ft^2.

Closing:

The area of a semicircular region is $\frac{1}{2}$ of the area of a circle with the same radius.

The area of a quarter of a circular region is $\frac{1}{4}$ of the area of a circle with the same radius.

If a problem asks you to use $\frac{22}{7}$ for π, look for ways to use fraction arithmetic to simplify your computations in the problem.

Problems that involve the composition of several shapes may be decomposed in more than one way.

Problem Set:
(s.124 -125)