Regents Exam in Algebra I (Common Core)

Materials for the May 2013 NTI

NOTE: Unless otherwise stated, all materials are draft. Finalized versions of these materials will be available on EngageNY or the NYSED Web site in Fall 2013. Please direct any questions or suggestions to the Office of State Assessment at emscassessinfo@mail.nysed.gov.
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About the Regents Examination in Algebra I (Common Core)

Designed to Measure the Common Core

Every question on the Regents Examination in Algebra I (Common Core) will measure the Common Core. Some questions are designed to measure a single standard; however, because of the integrated nature of the Common Core high school standards, most questions will measure more than one standard. In some cases a question will be designed to measure a full cluster or domain, rather than a single standard.

Students Who Focus on the Major Clusters and More Critical Standards Will Likely Do Better on the Exam

Not all standards are created equal in the Common Core; some standards are more important to the major work of the Algebra I course, while other standards are less critical to students’ understanding of the Algebra I content and the content of future courses in high school and college. The Regents Examination in Algebra I (Common Core) is intentionally constructed so that questions associated with those clusters specified in the PARCC Model Content Frameworks for Mathematics as having greater emphasis will appear more often on the exam than questions associated with less emphasized clusters. Additionally, NYSED has worked with New York State educators to determine which specific standards are more critical to a student’s success in Algebra I, Algebra II, future college math courses, and the math needed to succeed in careers. Those standards that have been determined to be more important will also have more questions written to them.

Standards Are Measured in Various Ways

In working with high school and college educators in New York State to build the Regents Examination in Algebra I (Common Core), it is clear that each standard should not be tested in exactly the same way on every exam. This approach to assessment is not indicative of what students experience in high school and college classrooms, nor is it representative of the actual mathematical problems that students will be confronted with in their careers and life experiences. With the new Algebra I (Common Core) exam, questions are written in different ways for each standard. These “multiple representations” of each standard will not only ensure that students are learning the content and practices demanded by the Common Core, but will also help to dissuade teachers from repeatedly drilling students on the same test preparation questions.

The Exam Will Assess the Key Skills and Content that Students Need to Be on Track for College and a Career

NYSED worked extensively with educators from SUNY, CUNY, and a number of independent colleges in New York State to ensure that the new Regents Examination in Algebra I (Common Core) measures the knowledge and skills needed for students to succeed in their first year of
credit-bearing college courses. For example, as a result of input from college educators, the Algebra I (Common Core) Exam will feature a number of questions built specifically to measure the Mathematical Practices specified by the Common Core. In most cases, these practices require complex mathematical thinking and test questions featuring real-world situations and are not the type of question seen on prior mathematics Regents Exams.

**Students Who Score Proficient on the Exam Will Be Ready for the Regents Examination in Algebra II (Common Core)**

The Regents Examination in Algebra I (Common Core) was intentionally co-developed with the Regents Examination in Algebra II (Common Core) to ensure that students who succeed on the Algebra I (Common Core) examination will be prepared to access the content featured on the Algebra II (Common Core) examination. Likewise, when NYSED sets the cut scores on the Algebra I (Common Core) examination, this will be done with the input of New York State educators who specialized in Algebra I, Algebra II, and college faculty members who teach entry-level college mathematics courses.
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Test Blueprint: Regents Examination in Algebra I (Common Core)

The Regents Examination in Algebra I (Common Core) will consist of 86 total credits (points) spread across 37 items. All items will measure the Common Core Algebra 1 standards as defined by the “Traditional Pathway” specified in the PARCC Model Content Frameworks for Mathematics (see http://www.p12.nysed.gov/assessment/math/ccmath/parccmcf.pdf for more information).

One of the major curricular shifts demanded by the Common Core for mathematics is a focus on the major work of each course. That is, not all content in a given course is emphasized equally in the standards. As noted in the PARCC Model Content Frameworks for Mathematics, “Focus is critical to ensure that students learn the most important content completely, rather than succumb to and overly broad survey of content.” In Algebra I, the major work of the course includes those standards associated with the “Algebra” and “Functions” conceptual categories. Additionally, the PARCC Model Content Frameworks for Mathematics specifies which clusters within the Common Core mathematics standards should require more focus during the academic year. As stated in the PARCC Model Content Frameworks for Mathematics, “Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness.”

In creating the Regents Examination in Algebra I (Common Core), NYSED has built an exam that allows educators to teach students the major work of the course and to focus on those standards associated with clusters that require the greatest emphasis during the academic year.

The test blueprint for the Regents Examination in Algebra I (Common Core) demonstrates NYSED’s commitment to ensuring that educators are able to focus their instruction on the most critical elements of the Algebra I course. Note, for example, that the majority of the points on the exam are associated with the Algebra and Functions conceptual categories, and that relatively few points are associated with the Number and Quantity and Statistics and Probability conceptual categories.
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### Test Blueprint: Regents Examination in Algebra I (Common Core)

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<tr>
<th>Conceptual Category</th>
<th>Percent of Exam by Points</th>
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<tr>
<td>Number and Quantity</td>
<td>2% - 8%</td>
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<tr>
<td>Algebra</td>
<td>50% - 56%</td>
</tr>
<tr>
<td>Functions</td>
<td>32% - 38%</td>
</tr>
<tr>
<td>Statistics and Probability</td>
<td>5% - 10%</td>
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New York State Common Core Algebra I Standards Clarifications

In January 2011, the NYS Board of Regents adopted the NYS P-12 Common Core Learning Standards (CCLS), which include the Common Core State Standards and a small number of additional unique standards added by New York State. The CCLS were created through a collaborative effort on behalf of the National Governor’s Association Center for Best Practices and the Council of Chief State School Officers. The standards were developed by key stakeholders in the field, including teachers, school administrators, and content experts.

The main design principles in the NYS CCLS for Mathematics standards are focus, coherence, and rigor. These principles require that at each grade level, students and teachers direct their time and energy on fewer topics in order to form deeper understandings, gain greater skill and fluency, and more robustly apply what is learned.

A Story of Functions, the curriculum overview for grades 9-12 found on the EngageNY website (http://www.engageny.org), provides an overview of the academic year for grades 9 through 12 including curriculum maps and detailed grade-level descriptions. The footnotes in this document help to clarify some standards and, in cases where a standard is shared between Algebra I and Algebra II, indicate what is appropriate for the Algebra I level. In an effort ensure that the standards can be interpreted by teachers and effectively utilized to inform classroom instruction, several additional standards of the Algebra I curriculum have been identified as needing some clarification. These clarifications are outlined below.
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Clarifications

N.Q.3
The greatest accuracy for a result is only at the level of the least accurate data point (example: if units are tenths and hundredths, then the appropriate accuracy level is tenths).

Calculation of relative error is not included in this standard.

A.SSE.1a
The “such as” listed are not the only parts of an expression students are expected to know; others include, but are not limited to, degree of a polynomial, leading coefficient, constant term, and the standard form of a polynomial (descending exponents).
A.SSE.2
Does not include factoring by grouping and factoring the sum and difference of cubes.

A.SSE.3a
Includes trinomials with leading coefficients other than 1.

A.REI.4
Solutions may include simplifying radicals.

S.ID.6a
Includes the use of the regression capabilities of the calculator.

S.ID.6b
Includes creating residual plots using the capabilities of the calculator (not manually).

S.ID.6c
Both correlation coefficient and residuals will be addressed in this standard.
Question Formats

The 2014 Regents Examination in Algebra I (Common Core) contains multiple-choice, 2-point constructed-response, 4-point constructed-response, and 6-point constructed-response questions. For multiple-choice questions, students select the correct response from four answer choices. For constructed-response questions, students write an answer to an open-ended question and may be required to show their work. In some cases, they may be required to explain, in words, how they arrived at their answers.

Multiple-Choice Questions

Multiple-choice questions are designed to assess CCLS for Mathematics. Mathematics multiple-choice questions will mainly be used to assess standard algorithms and conceptual standards. Multiple-choice questions incorporate both Standards and Standards for Mathematical Practices, some in real-world applications. Many multiple-choice questions require students to complete multiple steps. Likewise, many of these questions are linked to more than one standard, drawing on the simultaneous application of multiple skills and concepts. Within answer choices, distractors (i.e., an incorrect response that may appear to be a plausible correct response to a student who has not mastered the skill or concept being tested) will all be based on plausible missteps.

Constructed-Response Questions (2-, 4-, and 6-point)

Constructed-response questions require students to complete a task and show their work. Like multiple-choice questions, constructed-response questions will often require multiple steps, the application of multiple mathematics skills, and real-world applications. These questions allow students to show their understanding of mathematical procedures and demonstrate conceptual understanding. Many questions will cover conceptual and application standards. Constructed-response questions may also assess student reasoning and the ability to critique the arguments of others.
Question Types

The Regents Examination in Algebra I, Geometry, and Algebra II (Common Core) will include the following types and numbers of questions:

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<th>Question Type</th>
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<tr>
<td>2-credit open-ended</td>
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<tr>
<td>4-credit open-ended</td>
<td>4</td>
</tr>
<tr>
<td>6-credit open-ended</td>
<td>1</td>
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<td>Total credits</td>
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Guidelines for Writing Math Items

With the transition to the New York P-12 Common Core State Standards (CCSS), significant changes have been made to the New York State Testing Program (NYSTP). The 2014 Regents Examination in Algebra I (Common Core) will measure the CCSS for Mathematics. This test will approach Mathematics differently from past assessments.

To help ensure consistent and rigorous interpretation and measurement of the CCSS for Mathematics, the state has articulated criteria for developing test questions (items). Item writing is a crucial aspect of test development. These guidelines for writing multiple-choice and constructed-response items serve to ensure that the items included on operational exams meet certain standards for alignment to curriculum, fairness, clarity, and overall quality.

How the Item Writing Guidelines Are Used

Using these guidelines to draft questions is one of many steps employed to help ensure a valid, fair, and quality assessment. Draft questions that meet these criteria are allowed to move forward in the development process. The next step is for the items to be reviewed, and edited when necessary, by a Committee of certified New York State educators [1]. Only items that are approved by the educator panel are allowed to be field-tested.


How Educators Can Use the Item Writing Guidelines to Design Assessments or Modify Instruction

The Item Writing Guidelines can also serve as a helpful tool as educators develop assessment items and instructional activities. In order to ensure that assessment items and instructional activities align with the CCSS, they must adhere to the following for all items, while additional, format-specific guidelines apply to multiple-choice and constructed-response items:

1. The item should focus primarily on one learning standard.
2. The focus of the problem or topic should be stated clearly and concisely.
3. Include problems that come from a real-world context or problems that make use of multiple representations.
4. The item should be written with terminology, vocabulary and sentence structure kept as simple as possible. The item should be free of irrelevant or unnecessary detail.
5. The item should not contain clues that are extraneous to the correct answer.
6. The item should assess student understanding of the standards by requiring responses that show evidence of comprehension, application, analysis, synthesis, and/or evaluation.

7. The item should require work rather than just recall.

8. The stimulus should provide information/data that is mathematically accurate.

9. Items may be broken into multiple parts that may be labeled $a$, $b$, $c$, etc.

10. Symbolism as presented in the core curriculum and on previous examinations should be used consistently.
Guidelines for Writing Multiple-Choice Math Items

1. The item should focus primarily on one learning standard.
   When measuring more than one learning standard, specify the primary learning standard first and then all applicable learning standards.

2. The focus of the problem or topic should be stated clearly and concisely.
   The stem should be meaningful and convey the central problem. A multiple-choice item functions most effectively when a student is required to compare specific alternatives related to the stem. It should not be necessary for the student to read all of the alternatives to understand an item. *(Hint: Cover the alternatives and read the stem on its own. Then ask yourself if the question includes the essential elements or if the essential elements are lost somewhere in the alternatives.)*

3. Include problems that come from a real-world context or problems that make use of multiple representations.
   When using real-world problems, use formulas and equations that are real-world (*e.g.*, the *kinetic energy of an object with mass, m, and velocity, V is k = \( \frac{1}{2} mv^2 \))*). Use real-world statistics whenever possible.

4. The item should be written in clear and simple language, with vocabulary and sentence structure kept as simple as possible.
   Each multiple-choice item should be specific and clear. The important elements should generally appear early in the stem of an item, with qualifications and explanations following. Difficult and technical vocabulary should be avoided, unless essential for the purpose of the question.

5. The stem should be written as a direct question or an incomplete statement
   Direct questions are often more straightforward. However, an incomplete statement may be used to achieve simplicity, clarity, and effectiveness. Use whichever format seems more appropriate to present the item effectively.

6. The stem should not contain irrelevant or unnecessary detail.
   Be sure that sufficient information is provided to answer the question, but avoid excessive detail or “window dressing.”

7. The stem should be stated positively. Avoid using negatively stated stems.
   In general, avoid negative stems that may confuse the student. Items should never contain a double negative.
8. The phrase *which of the following* should not be used to refer to the alternatives; instead, use *which followed by a noun*.
   In the stem, *which of the following* requires the student to read all of the alternatives before knowing what is being asked and assessed. Expressions such as *which statement, which expression, which equation, and/or which graph* are acceptable.

9. The stem should include any words that must otherwise be repeated in each alternative.
   In general, the stem should contain everything the alternatives have in common or as much as possible of their common content. This practice makes an item concise. Exceptions include alternatives containing units and alternatives stated as complete sentences.

10. The item should have one and only one correct answer.
    Items should not have two or more correct alternatives. *All of the above and none of the above* are not acceptable alternatives.

11. The distracters should be plausible and attractive to students who lack the knowledge, understanding, or ability assessed by the item.
    Distracters should be designed to reflect common errors or misconceptions of students.

12. The alternatives should be grammatically consistent with the stem.
    Use similar terminology, phrasing or sentence structure in the alternatives. Alternatives must use consistent language, including verb tense, nouns, singular/plurals, and declarative statements. Place a period at the end of an alternative *only* if the alternative by itself is a complete sentence.

13. The alternatives should be parallel with one another in form.
    The length, complexity and specificity of the alternatives should be similar. For example, if the stem refers to a process, then all the alternatives must be processes. Avoid the use of absolutes such as *always and never* in phrasing alternatives.

14. The alternatives should be arranged in logical order, when possible.
    When the alternatives consist of numbers and letters, they should ordinarily be arranged in ascending or descending order. An exception would be when the number of an alternative and the value of that alternative are the same. For example: (1) 1 (2) 2 (3) 0 (4) 4.

15. The alternatives should be independent and mutually exclusive.
    Alternatives that are synonymous or overlap in meaning often assist the student in eliminating distracters.

16. The item should not contain extraneous clues to the correct answer.
Any aspect of the item that provides an unintended clue that can be used to select or eliminate an alternative should be avoided. For example, any term that appears in the stem should not appear in only one of the alternatives.

17. Symbols as presented in the core curriculum and on previous examinations should be used consistently.

For example, $AB$ means the length of line segment $AB$, $\overline{AB}$ means segment $AB$, $m\angle A$ means the number of degrees in the measure of angle $A$, etc.
Guidelines for Writing Open-Ended Math Items

1. **The item should focus primarily on one learning standard.**
   When measuring more than one learning standard, specify the primary learning standards first and then all applicable learning standards.

2. **The focus of the problem or topic should be stated clearly and concisely.**
   The item should be meaningful, address important knowledge and skills, and focus on key concepts.

3. **Include problems that come from a real-world context or that make use of multiple representations.**
   When using real-world problems, use formulas and equations that are real-world (e.g., the kinetic energy of an object with mass, \( m \), and velocity, \( V \) is \( k = \frac{1}{2} mv^2 \)). Use real-world statistics whenever possible.

4. **The item should be written with terminology, vocabulary and sentence structure kept as simple as possible. The item should be free of irrelevant or unnecessary detail.**
   The important elements should generally appear early in the item, with qualifications and explanations following. Present only the information needed to make the context/scenario clear.

5. **The item should not contain extraneous clues to the correct answer.**
   The item should not provide unintended clues that allow a student to obtain credit without the appropriate knowledge or skill.

6. **The item should assess student understanding of the standards by requiring responses that show evidence of comprehension, application, analysis, synthesis, and/or evaluation.**
   The item should require students to demonstrate depth of understanding and higher-order thinking skills through written expression, numerical evidence, and/or diagrams. An open-ended item should require more than an either/or answer or any variation such as yes/no, decrease/increase, and faster/slower. Often either/or items can be improved by asking for an explanation.

7. **The item should require work rather than just recall.**
   Students need to show their mathematical thinking in symbols or words.

8. **The stimulus should provide information/data that is mathematically accurate.**
   Examples of stimuli include, but are not limited to, art, data tables, and diagrams. It is best to use actual data whenever possible. Hypothetical data, if used, should be
plausible and clearly identified as hypothetical.

9. The item should be written so that the student does not have to identify units of measurement in the answer, unless the question is testing dimensional analysis. For example, consider the question: “A circle has a radius of length 4 centimeters. Find the number of centimeters in the length of the arc intercepted by a central angle measuring 2 radians.” Students would receive credit for an answer of “8” and would not be penalized for writing “8 cm.”

10. The item should be written to require a specific form of answer. Phrases like “in terms of \( \pi \),” “to the nearest tenth,” and “in simplest radical form” may simplify the writing of the rubric for these types of items.

11. Items that require students to explain in words are encouraged. One of the emphases of the new learning standards is to foster student ability to communicate mathematical thinking.

12. Items may be broken into multiple parts that may be labeled \( a \), \( b \), \( c \), etc. Clear division of the parts of the problems may simplify the writing of the rubric for these types of items.

13. Symbolism as presented in the core curriculum and on previous examinations should be used consistently. For example, \( AB \) means the length of line segment \( AB \), \( \overline{AB} \) means segment \( AB \), \( m \angle A \) means the number of degrees in the measure of angle \( A \), etc.
Multiple Representations

With the transition to the New York P12 Common Core State Standards (CCSS), significant changes have been made to the New York State Testing Program (NYSTP). The 2014 Regents Examination in Algebra I (Common Core) will measure the CCSS for Mathematics.

The CCSS for Mathematics strongly emphasize that students gain deep conceptual understanding as well as procedural proficiency in math. Likewise, the CCSS stress that students’ procedural knowledge and conceptual understandings are developed and demonstrated through application and modeling. As such, it is crucial that in both instruction and assessment, students are required to move far beyond simple, algorithmic strategies to solve math problems.

Predictable math questions (items) that require only algorithmic strategies to be solved correctly are commonplace in assessments. In an effort to ensure that the New York State Testing Program (NYSTP) measures the deep conceptual understanding that CCSS demands, the specifications for the design of assessment items include multiple representations.

Multiple Representations

Multiple Representations (MRs) are a broad set of specifications that describe, refer and symbolize the various, but not all, ways that math standards could be measured within the constraints of NYSTP. The MR document specifies three overarching families of MRs:

1. **Procedural Skills**: Procedural skills representations specifically apply to standards that reference verbs such as compute, solve, identify, interpret, use, make and find solutions. Procedural representations are most often multiple-choice questions that require students to apply and identify mathematical processes in various ways.

2. **Conceptual Understanding**: Conceptual understanding representations are applied to standards using verbs such as understand, explain, represent and describe. As a result, these items require different combined mathematical practices depending on the given item type or item.

3. **Application**: Application standards and items are unique within the Common Core. There are standards that reference application, which are represented by application tasks. Also, there are application tasks that are used to represent standards for which application is not explicitly required. Broadly speaking, application items require students to apply both procedural knowledge and conceptual understanding to complete a task.

Each family of MRs is explained in the document. The explanation includes a statement of how to identify standards that might be measured using the particular MR. The explanation also goes on to identify the types of math skills (e.g., application of process, explanation of a principle, etc.) that are appropriate to assess given that MR. A sample from the MR document has been included and annotated below.
Conceptual Understanding Item Formats

Conceptual Understanding representations are applied to standards using verbs including understand, explain, represent and describe. The resulting item types require different combined mathematical practices depending on the given item type or item.

C1) Explanation of Process/Principle/Rule: These types of items require students to identify or explain a component of math or mathematic understanding. Items that successfully assess conceptual understanding with this type of representation can be formatted in at least the following ways: (meaning)
   C1a. Verbal explanation of process/principle/rule
   C1b. Verbal explanation of result of process/principle/rule
   C1c. Verbal description of undoing the process/principle/rule.
   C1d. Identification of missing steps.
   C1e. Show meaning of procedure, strategy, or reasoning process, using examples.
   C1f. Show meaning of procedure using counterexamples.
   C1g. Explain errors in process/principle/rule.
   C1h. Explanation of choice in process/principle/rule.
   C1i. Select the most appropriate strategy to represent information
   C1j. Undue Formulic Strategies
   C1k. Differentiate between strategies

C2) Patterns and Relationships: These types of items require students to identify, define, and create patterns and relationships using numeric, abstract, geometric, and graphical information. Item formats that deal with patterns and relationship may include:

C2a. Generate similarities and differences between processes, graphs, tables, and patterns
C2b. Draw connections between numerical concepts and geometric scenarios
C2c. Explain the relationship between different numerical representations
C2d. Discern and articulate patterns represented numerically, graphically, or with

How NY Uses Multiple Representations for Assessment

The MRs articulate the various ways to develop math questions. As such, the MRs offer item developers guidance on interpreting the standards, the focus of measurement for each standard, and ways to measure different aspects of each standard through different approaches. Item developers are encouraged to use different formats to measure a single standard in order to more faithfully measure the whole intent of each standard. The MR approach helps NY to ensure that the assessments measure more than simple algorithmic strategies.
How Instructors Can Use Multiple Representations for Classroom Instruction

The MRs can be used to help an educator plan instruction with a variety of different approaches to the standard in mind in order to teach to the whole standard, as referenced above. Knowing that assessment items, over time, will assess a given standard through multiple formats, educators should approach the instruction of a given standard through multiple formats and perspectives. However, the state assessments do have their limitations. Instruction should not be limited to only those formats that fit within the constraints of large-scale assessment.

When planning instruction for a given standard, instructors should think about all of the multiple perspectives from which a standard can be interpreted, which means that instruction should approach standards from a:

- Conceptual,
- Procedural, and
- Application lens (family of item formats).

This type of thorough instruction will lead to foundational student understanding of each CCSS. This will enable students to apply their understanding to all of the specific formats listed in the MR document. Ultimately, teaching with the MR approach results in instruction that is more holistic. Student understanding becomes less about simple mastery and more about application of that understanding in a variety of ways. Instructors can access the curriculum modules available at http://engageny.org for guidance on developing holistic performance-based and classroom assignments.
Multiple Representations

Procedural Skills Item Formats

Procedural representations specifically apply to standards that reference the verbs including *compute, solve, identify, interpret, use, make and find solutions*. Procedural representations are most often MCQs that require students to apply and identify mathematic processes in various ways.

**P)** These items are conceptualized so that students:

Pa. Solve problems that involve forward computation, procedures, or process involving

Pb. Solve problems that involve non-forward computation, procedures, or process involving

Pc. Solve problems using numbers organized in different ways (horizontally, vertically in lists, tables)

Pd. Interpret and solve problems requiring the manipulation of numbers among different forms or from one form to another

Pe. Relate or evaluate two or more pieces of information to solve a problem vs. more directed stems not relating multiple pieces of information found within the problem context

Pf. Make or interpret conditional statements: “If... then...” (Student could identify example or create)

Pg. Identify extraneous information

Ph. Identify missing information that is required in order to fulfill claim of stem

Pi. Incorporate new information introduced in stem to information given in accompanying table, graph, or picture

Pj. Find values that make a process/principle/rule or relationship true

Pk. Modify, manipulate, or mark provided graphic to demonstrate concept or respond to claim in stem

Pl. Use provided values to demonstrate a process/principle/rule

Pm. Use symbols as placeholders to capture or emphasize the process/principle/rule

Pn. Recognize repetition of calculations and make use of shortcuts

Po. Illustrate/demonstrate the correct process/principle/rule without calculating the answer (or sets of correct processes/principles/rules)

Pp. Identify and explain errors in procedures

Pq. Relate new information from stem into information given in table or graph
Conceptual Understanding Item Formats

Conceptual understanding representations are applied to standards using verbs including understand, explain, represent and describe. The resulting item types require different combined mathematical practices depending on the given item type or item.

C1) Explanation of Process/Principle/Rule: These types of items require students to identify or explain a component of math or mathematical understanding. Items that successfully assess conceptual understanding with this type of representation can be formatted in at least the following ways:
   C1a. Verbal explanation of process/principle/rule
   C1b. Verbal explanation of result of process/principle/rule
   C1c. Verbal description of undoing the process/principle/rule
   C1d. Identification of missing steps
   C1e. Show meaning of procedure, strategy, or reasoning process, using examples
   C1f. Show meaning of procedure using counterexamples
   C1g. Explain errors in process/principle/rule
   C1h. Explanation of choice in process/principle/rule
   C1i. Select the most appropriate strategy to represent information
   C1j. Undo (deconstruct) formulaic strategies
   C1k. Differentiate between strategies

C2) Patterns and Relationships: These types of items require students to identify, define, and create patterns and relationships using numeric, abstract, geometric, and graphical information. Item formats that deal with patterns and relationship may include:
   C2a. Generate similarities and differences between processes, graphs, tables, and patterns
   C2b. Draw connections between numerical concepts and geometric scenarios
   C2c. Explain the relationship between different numerical representations
   C2d. Discern and articulate patterns represented numerically, graphically, or with symbols
   C2e. Create patterns from a rule, concept, visual, or verbal description
   C2f. Make inferences using data

C3. Transforming and Connecting Models: These items require students interpret between multiple contextualized and decontextualized informational sources. Item formats that deal with transforming and connecting models include:
   C3a. Translate between visual models and verbal descriptions
   C3b. Use visual models to explain and represent numerical concepts
   C3c. Create problem scenarios that illustrate mathematical concepts, processes, and procedures
   C3d. Use mathematical processes, principles or rules to verify a model
   C3e. Justify choice of particular model to fulfill a particular task or model a situation
   C3f. Classify and organize information
Application Item Formats

Application standards and items are unique within the core. There can be standards that reference applications that are represented by application tasks, and applications tasks that are used to represent standards where application is not explicitly called for. Broadly speaking, application items require students to apply both procedural and conceptual knowledge to complete a task. More specifically, application items require students to:

- make assumptions and simplifications
- model situations
- select appropriate knowledge and processes
- validate answers as reasonable or make generalizations within the problem-solving context

To successfully represent standards that reference application OR represent standards through application, item formats include:

A1. Prediction: Items may require students to make assumptions, select appropriate information, and form conjectures.

A2. Planning: Items require students to organize and select salient information in order to plan and respond to multifaceted demand. Often students will need to simplify the problem and determine the reasonableness of their solutions in the context of the problem.

A3. Proof: These items are intended for students to make assumptions, consider simpler cases of the problem, and construct a logical argument. Items will require students to judge the reasonableness or coherence of their results.

A4. Create Models: Items require students to make assumptions and apply appropriate knowledge and process to create models or problem scenarios.

A5. Solve Models: Items require students to make assumptions and apply appropriate knowledge and process to create solve real-world issue/problems.

A6. Evaluate: Infer truth or fallacy of statements based on outcome of procedures using given and missing information.
REVIEW CRITERIA CHECKLIST FOR POTENTIAL MATH ITEMS

With the transition to the New York P-12 Common Core State Standards (CCSS), significant changes have been made to the New York State Testing Program (NYSTP). The 2014 Regents Examination in Algebra I (Common Core) will measure the CCSS for Mathematics. This test will approach Mathematics differently from past assessments. To help ensure consistent and rigorous interpretation and measurement of the CCSS for Mathematics, the state has articulated criteria for reviewing test questions (items).

Item Review Criteria

The Item Review Criteria framework helps assessment developers and reviewers ensure that the Common Core Regents Exam in mathematics measures the CCSS for Mathematics with high-quality questions. The Item Review Criteria articulate the four major item characteristics NYSED looks for in developing quality questions. NYSED uses the Item Review Criteria to ensure:

1. Language and Graphical Appropriateness
2. Sensitivity/Bias
3. Fidelity of Measurement to CCSS (Item Alignment)
4. Conformity to the Expectations for the Specific Item Types and Formats (i.e., multiple-choice questions, 2-point constructed-response questions, 4-point constructed-response questions, and 6-point constructed-response questions).

The Item Review Criteria are organized into these four categories. Each section includes pertinent questions that help reviewers determine whether or not an item is of sufficient quality. Within the first two categories, the headings Language Appropriateness, Sensitivity/Bias, and Math Art identify the basic components of quality assessment items. The criteria for Language Appropriateness are used to help ensure that students understand what is asked in each question and that the language in the question does not adversely affect a student’s ability to perform the required task. Likewise, the Sensitivity/Bias criteria are used to evaluate whether questions are unbiased, non-offensive, and not disadvantageous to any given subgroup(s). The Math Art criteria assess the appropriateness and clarity when graphics are used within questions.

The third category of the Item Review Criteria framework, Item Alignment, addresses how each item measures a given Mathematics standard. This criterion asks the reviewer to comment on key aspects of how the item addresses and calls for the skills demanded by the standards. Additionally, these criteria prompt reviewers to comment on how more than one standard is addressed by a given question.
The fourth category of the Item Review Criteria framework addresses the specific demands for different item types and formats. Reviewers evaluate each item to ensure that it conforms to the given requirements. For example, multiple-choice items must have, among other characteristics, one unambiguous correct answer and plausible but incorrect answer choices.

How the Item Review Criteria Are Used

Using these criteria to review draft questions is one of many steps employed to help ensure a valid, fair, and quality assessment. Draft questions that meet these criteria are allowed to move forward in the development process. The next step is for the items to be reviewed, and edited when necessary, by a Committee of certified New York State educators [1]. The educator panel evaluates each question for fairness, valid measurement of the intended standard(s), and language clarity. Only items that are approved by the educator panel are allowed to be field-tested.


How Educators Can Use the Item Review Criteria to Design Assessments or Modify Instruction

The Item Review Criteria can also serve as helpful guidelines as educators develop assessment items and instructional activities. Teachers will want to consider:

1. Language and Graphical Appropriateness
2. Sensitivity/Bias
3. Fidelity to the precise language and structure of the CCSS
   1. In order to ensure the assessment items and instructional activities align with the CCSS, they must adhere to the following:
      1. The learning targets for a lesson are simply the standards themselves or portions of the standards themselves.
      2. The assessment question prompts/instructional materials use language from the actual standard(s) or the alignment is easily discernible.
      3. The learning experience directly informs students' understanding/the item requires students to show understanding of aspects of the particular standard(s).
      4. The item/learning experience does not require students to attend to all aspects of the standard(s).
      5. The item/learning experience is dependent on the students' ability to access skills referenced in the primary standard as well as the secondary and additional standards.
6. The item/learning experience measures/teaches students to show understanding of the skills in the standard. The item/learning experience includes nonstandard numbers (e.g., students are asked to solve questions using non-whole numbers).

4. Mirroring in item/lesson design to the "Expectations for the Specific Item Types and Formats" (i.e., multiple-choice questions, 2-point constructed-response questions, 4-point constructed-response questions, and 6-point constructed-response questions).
REVIEW CRITERIA CHECKLIST FOR POTENTIAL MATH ITEMS

The following list of criteria will be used to train item writers and then to review items for possible inclusion on test forms.

<table>
<thead>
<tr>
<th>Language Appropriateness</th>
<th>Yes</th>
<th>No</th>
<th>n/a</th>
<th>Explain or Describe</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Item:</td>
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<tr>
<td>Uses grade-level vocabulary.</td>
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<td>Uses the simplest terms possible to convey information.</td>
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<td>Avoids technical terms unrelated to content.</td>
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<td>2. Sentence complexity well within grade expectations.</td>
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<tr>
<td>3. Avoids ambiguous or double-meaning words.</td>
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<tr>
<td>4. Pronouns have clear referents.</td>
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<tr>
<td>5. Item avoids irregularly spelled words.</td>
<td>Yes</td>
<td></td>
<td>n/a</td>
<td>Use most common spelling of words</td>
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<tr>
<td>6. Item can be put into Braille.</td>
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<td>Item can be translated appropriately according to the specific accommodations as outlined in universal design guidelines</td>
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</tbody>
</table>


<table>
<thead>
<tr>
<th>Sensitivity/Bias</th>
<th>Yes</th>
<th>No</th>
<th>n/a</th>
<th>Explain or Describe</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The item is free of content that might be deemed offensive to groups of students, based upon culture, religion, race, ethnicity, gender, geographic location, ability, socioeconomic status, etc.</td>
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<tr>
<td>2. The item is free of content that contains stereotyping.</td>
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<tr>
<td>3. The item is free of content that might unfairly advantage or disadvantage subgroups of students (ethnicity, gender, geographic location, ability, socioeconomic status, etc.) by containing unfamiliar contexts or examples, unusual names of people or places, or references to local events or issues.</td>
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<tr>
<td>Math Art</td>
<td>Yes</td>
<td>No</td>
<td>n/a</td>
<td>Explain or Describe</td>
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</tr>
<tr>
<td>1. The artwork clearly relates to the item and is important as an aspect of the problem-solving experience.</td>
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<td>n/a</td>
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<tr>
<td>2. The details in the artwork accurately and appropriately portray numbers/concepts contained in text or in lieu of text. <em>Items should be drawn to scale as much as possible. By default, we do not include the text “Not drawn to scale” on every item; however, if a figure is drawn and there is a distortion in the figure, it should be indicated under the art that the figure is “not drawn to scale.” The degree of distortion should not be actively misleading.</em></td>
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<td>n/a</td>
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<tr>
<td>3. Graphics are clear (symbols are highly distinguished, free from clutter, at a reasonable scale, etc.)</td>
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<td>n/a</td>
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<tr>
<td>4. Visual load requirements are reasonable (interpreting graphic does not confuse underlying construct) and as simple as possible to present the prompt. <em>“Visual load” refers to the amount of visual/graphic material included within a contained space. When graphics become overly busy, they break the cognitive process for different people or trip people up.</em></td>
<td></td>
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<td>n/a</td>
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<tr>
<td>Item Alignment</td>
<td>Yes</td>
<td>No</td>
<td>n/a</td>
<td>Explain or Describe</td>
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<tr>
<td>1. Is the item aligned to the standard to which it is written?</td>
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<td><strong>List the primary standard to which the item is aligned and explain the degree to which there is alignment/lack of alignment.</strong></td>
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<tr>
<td>2. Is the item aligned to the correct secondary/tertiary standard(s)?</td>
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<td>3. The stem is reflective of the concept embedded within the standard and is representative of the goal of the standard.</td>
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<tr>
<td>4. The item requires students to show understanding of key aspects of the standard. <strong>If “No,” which aspects are not attended to?</strong> For constructed response items, it is important that the item be solved through an understanding of the key point of the standard. For example, if the language of the standard calls for “prove” or “show,” items should actually involve proof to be aligned, not simply the ability to solve a related problem or perform a related manipulation.</td>
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<td>5. Does the question lend itself to being answered using a below-grade-level standard rather than the skills/concepts references in the on-grade-level standard?</td>
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</table>
6. The item requires the student to use skills referenced in the primary standard and any additional standards listed.

7. The item includes grade/course-appropriate standard numbers/variables (e.g., students are asked to solve questions using numbers/variables that are grade-appropriate).
   
   Note: This includes the parameters outlined in the PARCC Pathways document for guidance on how some standards are split across A1 and A2.

8. The item is aligned to the correct primary Multiple Representation(s).
   If “No,” indicate the correct MR code(s).

9. Does the item expect that students use a formula? If so, the formula should be in the item stem or on the reference sheet.
   
   For example, the formula for gravity from physics should be included in the item stem.
<table>
<thead>
<tr>
<th>Application/Modeling Items</th>
<th>Yes</th>
<th>No</th>
<th>n/a</th>
<th>Explain or Describe</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The item is aligned to a standard that requires modeling/application.</td>
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<tr>
<td>Note: See starred items in CCSS for high school math. These items are identified as lending themselves to modeling.</td>
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<tr>
<td>2. Does the language of the item obscure the math concept being assessed?</td>
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<tr>
<td>Students should not stumble over irrelevant information.</td>
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<tr>
<td>4. Modeling/application scenario is realistic and appropriate to the grade level (the situation is one that a reasonable person would encounter in everyday life—no stretching velvet ropes or weighing kittens in milligrams).</td>
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<td>If “No,” explain why it’s not.</td>
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<tr>
<td>5. Standard does not call for modeling/application, but there is a reason for it to be represented as such.</td>
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<tr>
<td>Even non-starred standards can and should involve appropriate applications where possible.</td>
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<tr>
<td>6. Figures/numbers/concepts used in modeling/application as well as in the response are realistic (e.g., downloads cost 99 cents, the side of a house isn’t 3x-32 long)</td>
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<td>7. Modeling scenario is presented in the most realistic and simple manner possible.</td>
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<td>8. Modeling/application scenario does not assume outside knowledge (e.g., approximate weight of paper, definition of a micron).</td>
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<tr>
<td>9. Modeling/application scenario provides all necessary information for student to apply math concepts.</td>
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<tr>
<td>10. Item does not clue students to which math strategy is needed to solve, but rather allows the student to choose a strategy to solve the item correctly.</td>
<td>For example, we should not tell students to use Pythagorean theorem, but rather allow them to decide which approach to solving is appropriate.</td>
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</tr>
<tr>
<td>Mathematic Correctness</td>
<td>Yes</td>
<td>No</td>
<td>n/a</td>
<td>Explain or Describe</td>
</tr>
<tr>
<td>------------------------</td>
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</tr>
<tr>
<td>1. The stem addresses a central math concept, either implicitly or explicitly.</td>
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<tr>
<td>2. The math presented in stem is clear, accurate, and conceptually plausible.</td>
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<tr>
<td>3. At least one strategy exists that is on grade level to solve the problem.</td>
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<tr>
<td>4. If there is more than one strategy, regardless of the strategy employed, the same correct answer will be achieved.</td>
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<tr>
<td>5. There is a rationale for the correct response that is aligned to the language of the Standards and that demonstrates knowledge and/or application of the Standards.</td>
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<tr>
<td>6. For MCQs: Is answer Choice 1 plausible or the correct answer?</td>
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<td>If not, why?</td>
</tr>
<tr>
<td>7. For MCQs: Is answer Choice 2 plausible or the correct answer?</td>
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<td></td>
<td>If not, why?</td>
</tr>
<tr>
<td>8. For MCQs: Is answer Choice 3 plausible or the correct answer?</td>
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<td></td>
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<td>If not, why?</td>
</tr>
<tr>
<td>9. For MCQs: Is answer Choice 4 plausible or the correct answer?</td>
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<td>If not, why?</td>
</tr>
<tr>
<td>Constructed Response and All Regents</td>
<td>Yes</td>
<td>No</td>
<td>n/a</td>
<td>Explain or Describe</td>
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<td>-------------------------------------</td>
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</tr>
<tr>
<td>1. The item involves a multi-step process.</td>
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</tbody>
</table>
| 2. The item requires students to show work.  
*Work referenced in item should not be trivial (e.g., if work was not shown, it would be likely that mistakes would be made).* |     |    |     |                     |
| 3. The item assesses more than computation. |     |    |     |                     |
| 4. The item asks student to explain a concept or procedure used to solve the problem.  
*Note: Not always applicable* |     |    |     |                     |
| 5. If students are asked to describe what they did, clear direction is given as to what they should describe (the theory, the rationale for the answer, the reason a strategy is wrong, etc.). |     |    |     |                     |
| 6. The item explicitly describes what we’re trying to elicit from the student. |     |    |     |                     |
| 7. The item is presented in a manner consistent with the Application MRs.  
*Explain which one, and how it is realized.* |     |    |     |                     |
<table>
<thead>
<tr>
<th>Overarching Comments</th>
<th>Yes</th>
<th>No</th>
<th>Explain or Describe</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The item is aligned to standard.</td>
<td></td>
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<tr>
<td>2. The item is rigorous.</td>
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<td><em>The math should be sound, tight, challenging, and at the appropriate level of difficulty.</em></td>
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<td><em>Note: Does not apply if at final typesetting phase</em></td>
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<tr>
<td>3. Reject.</td>
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</table>
Mathematics Tools for Regents Exams in Algebra I, Geometry, and Algebra II (Common Core)

Mathematics tools are necessary for students to meet the Standards for Mathematical Practice in the New York State P-12 Common Core Learning Standards for Mathematics. For example:

**Use appropriate tools strategically**
Mathematically proficient students consider the available tools when solving a mathematical problem. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

**Attend to precision**
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school, they have learned to examine claims and make explicit use of definitions.
Mathematics Tools for Regents Exams in Algebra I, Geometry, and Algebra II (Common Core)

Why Mathematics Tools?

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**Calculators and Rulers (straight edge)**
A graphing calculator and ruler (straight edge) must be available to all students taking the Regents Exams in Algebra I, Geometry, and Algebra II (Common Core).
Compasses
A compass must be available to all students taking the Regents Exam in Geometry (Common Core).

Note: Schools are responsible for supplying the appropriate tools for use with the Regents Exams in Algebra I, Geometry, and Algebra II (Common Core). NYSED does not provide them.

Value of Pi
Students should use the $\pi$ symbol and its corresponding value when applicable on the Regents Exams in Algebra I, Geometry, and Algebra II (Common Core). The approximate values of $\pi$, such as 3.1416, 3.14 or $\frac{22}{7}$, are unacceptable unless otherwise specified.
Reference Sheet

The formulas that students will need to answer test questions will either be provided as part of the Reference Sheet or within specific items of the test.

The reference sheet for the Regents Examination in Algebra I (Common Core) has been developed based on the intent of the Common Core State Standards for Mathematics. These reference sheets include formulas and reference information that students need to assist them in answering certain questions on the exam.

The measurement formulas provided on the reference sheet only include the name of the figure or object to which the measurement formula pertains. The intent of the Common Core State Standards in Mathematics is to know and apply the measurement formulas. In order for students to be able to choose the correct formula, they will need to know the formula.
## CONVERSIONS

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<tr>
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<tr>
<td>1 liter</td>
<td>1000 cubic centimeters</td>
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## FORMULAS

### Triangle

\[ A = \frac{1}{2}bh \]

### Pythagorean Theorem

\[ a^2 + b^2 = c^2 \]

### Parallelogram

\[ A = bh \]

### Quadratic Formula

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

### Circle

\[ A = \pi r^2 \]

### Arithmetic Sequence

\[ a_n = a_1 + (n-1)d \]

### Circle

\[ C = \pi d \text{ or } C = 2\pi r \]

### Geometric Sequence

\[ a_n = a_1 r^{n-1} \]

### General Prisms

\[ V = Bh \]

### Geometric Series

\[ S_n = \frac{a_1 - a_1 r^n}{1 - r} \text{ where } r \neq 1 \]

### Cylinder

\[ V = \pi r^2 h \]

### Radians

\[ 1 \text{ radian} = \frac{180}{\pi} \text{ degrees} \]

### Sphere

\[ V = \frac{4}{3} \pi r^3 \]

### Degrees

\[ 1 \text{ degree} = \frac{\pi}{180} \text{ radians} \]

### Cone

\[ V = \frac{1}{3} \pi r^2 h \]

### Exponential Growth/Decay

\[ A = A_0 e^{k(t-t_0)} + B_0 \]

### Pyramid

\[ V = \frac{1}{3} Bh \]
New York State Common Core Sample Questions
Regents Examination in Algebra I (Common Core)

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How to Use the Sample Questions

- Interpret how the standards are conceptualized in each question.
- Note the multiple ways the standard is assessed throughout the sample questions.
- Look for opportunities for mathematical modeling, i.e., connecting mathematics with the real world by conceptualizing, analyzing, interpreting and validating conclusions in order to make decisions about situations in everyday life, society, or the workplace.
- Consider the instructional changes that will need to occur in your classroom.
- Notice the application of mathematical ways of thinking to real-world issues and challenges.
- Pay attention to the strong distractors in each multiple-choice question.
- **Don’t** consider these questions to be the only way the standard will be assessed.
• Don't assume that the sample questions represent a mini-version of future State exams.

**Understanding Math Sample Questions**

**Multiple Choice**

Sample multiple-choice math questions are designed to assess CCLS math standards and incorporate both standards and math practices in real-world applications. Math multiple-choice questions assess procedural and conceptual standards. Unlike questions on past math exams, many require the use of multiple skills and concepts. Answer choices are also different from those on past exams. Within the sample questions, all distractors will be based on plausible missteps.

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Sample Question #1 vs. August 2011 - #15

15. The data in the table below are graphed, and the slope is examined.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>9.0</td>
</tr>
<tr>
<td>1</td>
<td>8.75</td>
</tr>
<tr>
<td>1.5</td>
<td>8.5</td>
</tr>
<tr>
<td>2</td>
<td>8.25</td>
</tr>
<tr>
<td>2.5</td>
<td>8.0</td>
</tr>
</tbody>
</table>

The rate of change represented in this table can be described as

(1) negative.  
(2) positive.  
(3) undefined.  
(4) zero.
24. The equation $y = x^2 + 3x - 18$ is graphed on the set of axes below.

Based on this graph, what are the roots of the equation $x^2 + 3x - 18 = 0$?

(1) -3 and 6
(2) 0 and -18
(3) 3 and -6
(4) 3 and -18
Sample Question #3 vs. January 2011 - #28

28. Which equation has roots of -3 and 5?

(1) $x^2 + 2x - 15 = 0$

(2) $x^2 - 2x - 15 = 0$

(3) $x^2 + 2x + 15 = 0$

(4) $x^2 - 2x + 15 = 0$
25. If \( n \) is an odd integer, which equation can be used to find three consecutive odd integers whose sum is -3?

(1) \( n + (n + 1) + (n + 3) = -3 \)
(2) \( n + (n + 1) + (n + 2) = -3 \)
(3) \( n + (n + 2) + (n + 4) = -3 \)
(4) \( n + (n + 2) + (n + 3) = -3 \)
Sample Question #5 vs. June 2011 - #35

35. Chelsea has $45 to spend at the fair. She spends $20 on admission and $15 on snacks. She wants to play a game that costs $0.65 per game. Write an inequality to find the maximum number of times, \( x \), Chelsea can play the game.

Using this inequality, determine the maximum number of times she can play the game.
37. The cost of 3 markers and 2 pencils is $1.80. The cost of 4 markers and 6 pencils is $2.90. What is the cost of each item? Include appropriate units in your answer.
36. A turtle and a rabbit are in a race to see who is first to reach a point 100 feet away. The turtle travels at a constant speed of 20 feet per minute for the entire 100 feet. The rabbit travels at a constant speed of 40 feet per minute for the first 50 feet, stops for 3 minutes, and then continues at a constant speed of 40 feet per minute for the last 50 feet.

Determine which animal won the race and by how much time.
35. The cost of three notebooks and four pencils is $8.50. The cost of five notebooks and eight pencils is $14.50. Determine the cost of one notebook and the cost of one pencil. [Only an algebraic solution can receive full credit.]

**Teacher note:** Students are allowed to find a solution using multiple pathways. Also, students now need to complete six point questions which include an increased level of depth of understanding, application and the ability to justify their solution.
August 2011 Item #15

Option 1 is correct. The rate of change of the data is $\frac{-1}{2}$ which is negative.

Teacher note: Students are required to apply their knowledge of slope and rate of change to multiple functions and representations within one question versus only one function as in the past.

June 2009 Item #24

Option 3 is correct. The graph of the polynomial intersects the x-axis at points (–6, 0) and (3, 0). These are the only points on the graph where $y = 0$.

Teacher note: Students are required to apply new vocabulary and the corresponding notation.

January 2011 Item #28

Option 2 is correct.

\[
\begin{align*}
  x &= -3 \text{ and } x = 5 \\
  x + 3 &= 0 \text{ and } x - 5 = 0 \\
  0 &= (x + 3)(x - 5) \\
  0 &= x^2 + 3x - 5x - 15 \\
  0 &= x^2 - 2x - 15
\end{align*}
\]

Teacher note: Students are required to apply new vocabulary (zeros) and the corresponding notation.

June 2012 Item #25

Option 3 is correct.

Let $n$ = the 1st odd integer
Then $n + 2$ = the 2nd consecutive odd integer and
$n + 4$ = the 3rd consecutive odd integer

Therefore, three consecutive odd integers that sum to –3 can be represented as

\[n + (n + 2) + (n + 4) = -3\]

Teacher note: In line with the fifth instructional shift (Application), students are required to apply two concepts such as consecutive integers and Pythagorean Theorem.
June 2011 Item #35

Let $x =$ # of games played

\[
45 \geq 20 + 15 + .65x \\
45 \geq 35 + .65x \\
10 \geq .65x \\
15.38 \geq x
\]

Chelsea can play a maximum of 15 games.

**Teacher note:** This question is an example of the increased rigor called for by the Common Core Learning Standards. Questions that had been worth 3 points may be worth only 2 points on a Common Core Regents Exam.

August 2008 Item #37

Let $m =$ the cost of one marker
Let $p =$ the cost of one pencil

\[
\begin{align*}
(3m + 2p = 1.80) \quad (-3) & \quad = \quad -9m - 6p = -5.40 \\
4m + 6p = 2.90 & \quad = \quad 4m + 6p = 2.90 \\
-5m & = 2.50 \\
m & = .50
\end{align*}
\]

\[
\begin{align*}
3m + 2p & = 1.80 \\
3(.50) + 2p & = 1.80 \\
1.50 + 2p & = 1.80 \\
2p & = .30 \\
p & = .15
\end{align*}
\]

One marker costs $0.50 and one pencil cost $0.15.

**Teacher note:** Students are required to not only solve a system of equations/inequalities, but may need to justify their solution as well.

January 2012 Item #36

Let $t =$ the turtle’s time

\[
t = \frac{100ft}{20ft/min} = 5 \text{ minutes}
\]
Let \( r \) = the rabbit’s time

\[
r = \frac{50 \text{ ft}}{40 \text{ ft/min}} + 3 \text{ minutes} + \frac{50 \text{ ft}}{40 \text{ ft/min}}
\]

\[
r = \frac{100 \text{ ft}}{40 \text{ ft/min}} + 3 \text{ minutes} = 2.5 \text{ minutes} + 3 \text{ minutes} = 5.5 \text{ minutes}
\]

The turtle won the race by 0.5 minutes.

**Teacher note:** Students are required to represent data in multiple ways and interpret their findings.

---

**January 2013 Item #35**

Let \( n \) = the cost of one notebook
Let \( p \) = the cost of one pencil

\[
(3n + 4p = 8.50) \quad (5) \quad = \quad 15n + 20p = 42.50
\]

\[
(5n + 8p = 14.50) \quad (-3) \quad = \quad -15n - 24p = -43.50
\]

\[
\begin{align*}
-4p &= -1 \\
p &= 0.25
\end{align*}
\]

\[
3n + 4p = 8.50 \\
3n + 4(0.25) = 8.50 \\
3n + 1 = 8.50 \\
3n = 7.50 \\
n = 2.50
\]

One pencil cost $0.15 and one marker costs $0.50.

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New York State Common Core Sample Questions: Regents Examination in Algebra I (Common Core)

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1. Given the functions $g(x)$, $f(x)$, and $h(x)$ shown below:

$$g(x) = x^2 - 2x$$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
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<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

The correct list of functions ordered from greatest to least by average rate of change over the interval $0 \leq x \leq 3$ is

(1) $f(x)$, $g(x)$, $h(x)$
(2) $h(x)$, $g(x)$, $f(x)$
(3) $g(x)$, $f(x)$, $h(x)$
(4) $h(x)$, $f(x)$, $g(x)$
2. The graphs below represent functions defined by polynomials. For which function are the zeros of the polynomials 2 and –3?

![Graphs](image_url)
3. For which function defined by a polynomial are the zeros of the polynomial –4 and –6?

(1) \( y = x^2 - 10x - 24 \)
(2) \( y = x^2 + 10x + 24 \)
(3) \( y = x^2 + 10x - 24 \)
(4) \( y = x^2 - 10x + 24 \)
4. The length of the shortest side of a right triangle is 8 inches. The lengths of the other two sides are represented by consecutive odd integers. Which equation could be used to find the lengths of the other sides of the triangle?

(1) \(8^2 + (x + 1) = x^2\)
(2) \(x^2 + 8^2 = (x + 1)^2\)
(3) \(8^2 + (x + 2)^2 = x^2\)
(4) \(x^2 + 8^2 = (x + 2)^2\)
5. Donna wants to make trail mix made up of almonds, walnuts and raisins. She wants to mix one part almonds, two parts walnuts, and three parts raisins. Almonds cost $12 per pound, walnuts cost $9 per pound, and raisins cost $5 per pound.

Donna has $15 to spend on the trail mix. Determine how many pounds of trail mix she can make. [Only an algebraic solution can receive full credit.]
6. A high school drama club is putting on their annual theater production. There is a maximum of 800 tickets for the show. The costs of the tickets are $6 before the day of the show and $9 on the day of the show. To meet the expenses of the show, the club must sell at least $5,000 worth of tickets.

a) Write a system of inequalities that represent this situation.

b) The club sells 440 tickets before the day of the show. Is it possible to sell enough additional tickets on the day of the show to at least meet the expenses of the show? Justify your answer.
During a snowstorm, a meteorologist tracks the amount of accumulating snow. For the first three hours of the storm, the snow fell at a constant rate of one inch per hour. The storm then stopped for two hours and then started again at a constant rate of one-half inch per hour for the next four hours.

a) On the grid below, draw and label a graph that models the accumulation of snow over time using the data the meteorologist collected.

b) If the snowstorm started at 6 p.m., how much snow had accumulated by midnight?
8. Next weekend Marnie wants to attend either carnival A or carnival B. Carnival A charges $6 for admission and an additional $1.50 per ride. Carnival B charges $2.50 for admission and an additional $2 per ride.

a) In function notation, write $A(x)$ to represent the total cost of attending carnival A and going on $x$ rides. In function notation, write $B(x)$ to represent the total cost of attending carnival B and going on $x$ rides.

b) Determine the number of rides Marnie can go on such that the total cost of attending each carnival is the same. [Use of the set of axes below is optional.]

c) Marnie wants to go on five rides. Determine which carnival would have the lower total cost. Justify your answer.
Item #1

Key: 4

Aligned to CCLS: F.IF.6

Mathematical Practices: 2

Commentary: This question aligns to F.IF.6 because it assesses a student’s ability to calculate the average rate of change of a function presented symbolically, as a table, and graphically.

Rationale: Option 4 is correct. Over the interval $0 \leq x \leq 3$, the average rate of change for $g(x) = \frac{3}{3} = 1$, $f(x) = \frac{6}{3} = 2$, and $h(x) = \frac{7}{3} = 2 \frac{1}{3}$. Ordering these values from greatest to least results in the list of functions: $h(x), f(x), g(x)$.

Item #2

Key: 3

Aligned to CCLS: A.APR.3

Commentary: This question aligns to A.APR.3 because it requires a student to identify the graph of a polynomial with two given zeros.

Rationale: Option 3 is correct. The graph of the polynomial intersects the $x$-axis at points $(-3, 0)$ and $(2, 0)$. These are the only points on the graph where $y = 0$.

Item #3

Key: 2

Aligned to CCLS: A.APR.3

Mathematical Practices: 2

Commentary: This question aligns to A.APR.3 because it requires a student to identify the equation of a polynomial with two given zeros.
**Rationale:** Option 2 is correct.

\[ x = -4 \] and \[ x = -6 \]
\[ x + 4 = 0 \] and \[ x + 6 = 0 \]
\[ 0 = (x + 4)(x + 6) \]
\[ 0 = x^2 + 4x + 6x + 24 \]
\[ 0 = x^2 + 10x + 24 \]

Item #4

**Key:** 4

**Aligned to CCLS:** A.CED.1

**Mathematical Practices:** 1 and 2

**Commentary:** This item aligns to A.CED.1 because the student creates an equation in one variable that can be used to solve a problem.

**Rationale:** Option 4 is correct.

**Item #5**

**Key:** 2 pounds of trail mix

**Aligned to CCLS:** A.CED.1

**Mathematical Practices:** 1 and 2

**Commentary:** This question aligns to A.CED.1 because the student creates equations in one variable and uses them to solve a problem.

**Rationale:** Let \( x \) = pounds of an ingredient. Then the number of pounds of trail mix is represented by the expression \( x + 2x + 3x \). Therefore, the number of pounds of trail mix is \( 6x \). Then,
12x + 9(2x) + 5(3x) = 15
45 = 15
x = \frac{1}{3}

So, 6\left(\frac{1}{3}\right) = 2 \text{ pounds.}

Rubric:

[2] 2 and appropriate work is shown.

[1] Appropriate work is shown, but one computational error is made, but an appropriate number of pounds is stated.

or

[1] Appropriate work is shown, but one conceptual error is made, but an appropriate number of pounds is stated.

or

[1] 2, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Item #6

Key: a) \(x + y \leq 800\)
\[6x + 9y \geq 5000\]

b) Yes with appropriate work shown to justify the answer.

Aligned to CCLS: A.CED.3

Commentary: This question aligns to A.CED.3 because a student writes a system of inequalities to determine a viable solution.

Mathematical Practices: 4 and 6

Rationale:

a) Let \(x\) = number of presale tickets
\(y\) = number of day of show tickets

\(x + y \leq 800\)
\(6x + 9y \geq 5000\)

b) \(6(440) + 9y \geq 5000\)
\(2640 + 9y \geq 5000\)
9y ≥ 2360
y ≥ 262.2

263 tickets

440 advance purchase tickets added to 263 day of show tickets is 703 tickets, which is below the 800 ticket maximum. So yes, it is possible.

Rubrics:

(a) [2] \( x + y \leq 800 \) and \( 6x + 9y \geq 5000 \).
   
   [1] \( x + y \leq 800 \) or \( 6x + 9y \geq 5000 \).  
   
   or

   [1] \( x + y = 800 \) and \( 6x + 9y = 500 \).
   
   [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(b) [2] Yes, and appropriate work is shown.

   [1] Appropriate work is shown, but “yes” is not stated.
   
   [1] Appropriate work is shown, but one computational error is made, but an appropriate determination is made.
   
   or

   [1] Appropriate work is shown, but one conceptual error is made, but an appropriate determination is made.
   
   [0] Yes, but no work is shown.
   
   or

   [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Item #7

Key: a) See graph in rationale below.

   
   b) \( 3 \frac{1}{2} \)

Aligned to CCLS: F.IF.4
Commentary: This question aligns to F.IF.4 because the students sketch a graph based on a verbal description of the snowstorm.

Mathematical practices: 4

Rationale:

Rubric:

[4] A correct graph is drawn, the axes are labeled correctly, and \( \frac{31}{2} \) is stated.

[3] Appropriate work is shown, but one graphing or labeling error is made, but an appropriate amount of snow is stated.

or

[3] A correct graph is drawn, the axes are labeled correctly, but the amount of snow is missing or is incorrect.

[2] Appropriate work is shown, but two or more graphing or labeling errors are made, but an appropriate amount of snow is stated.

or

[2] Appropriate work is shown, but one conceptual error is made, but an appropriate amount of snow is stated.

or

[2] Appropriate work is shown, but one graphing or labeling error is made, and the amount of snow is missing or is incorrect.

[1] Appropriate work is shown, but two or more graphing or labeling errors are made, and the amount of snow is missing or incorrect.

or
[1] Appropriate work is shown, but one conceptual error and one graphing or labeling error are made, but an appropriate amount of snow is stated.

or

[1] Appropriate work is shown, but one conceptual error is made, and the amount of snow is missing or is incorrect.

or

[1] $3\frac{1}{2}$, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Item #8

Key: a) $A(x) = 1.50x + 6$
    $B(x) = 2x + 2.50$

b) 7 rides

c) Carnival B with appropriate justification.

Aligned to CCLS: A.REI.11

Mathematical Practices: 2, 3, and 4

Commentary: This question aligns to A.REI.11 because the answer to the problem requires the student to solve $A(x) = B(x)$, either algebraically or graphically.

Rationale:

a) $A(x) = 1.50x + 6$
    $B(x) = 2x + 2.50$

b) $A(x) = B(x)$
    $1.50x + 6 = 2x + 2.5$
    $x = 7$

c) Carnival A cost = $1.50x + 6$
    $= 1.50(5) + 6$
    $= $13.50

    Carnival B cost = $2x + 2.50$
    $= 2(5) + 2.50$
    $= $12.50

Carnival B because it costs $12.50 and carnival A costs $13.50.
**Rubrics:**

(a)  [2] \( A(x) = 1.50x + 6 \) and \( B(x) = 2x + 2.50 \)

[1] Either \( A(x) = 1.50x + 6 \) or \( B(x) = 2x + 2.50 \) is written.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(b)  [2] 7 and appropriate work is shown.

[1] Appropriate work is shown, but one computational or graphing error is made, but an appropriate number of rides is stated.

or

[1] Appropriate work is shown, but one conceptual error is made, but an appropriate number of rides is stated.

or

[1] 7, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(c)  [2] Carnival \( B \) and an appropriate justification is given, such as showing that carnival \( B \) costs $12.50 and carnival \( A \) costs $13.50.

[1] Carnival \( B \), but the justification is incomplete or incorrect.

[0] Carnival \( B \), but no explanation is given.
CCSS.Math.Content.7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.
1. An online magazine charges companies to place advertisements on its web site. Each web page is divided into a grid to sell advertisements in the regions labeled below:

The prices are constantly changing so the following ratios are used to calculate charges for advertisers. The cost of advertising in region A compared to region B is 3:2, the cost of region A to region C is 3:1 and the cost of region A to region D is 4:1.

If region A is currently selling for $600 a month, how much more does an advertisement cost in region B compared to region D?

A $150  
B $250  
C $400  
D $450  

Key: B

Commentary: This item aligns to 7.RP.A.3 because it involves a series of ratios that determine how the advertisements on a web page are priced. As a result students must use proportions to solve a multistep ratio problem.
**Multiple Representations:** Solving this item requires that students correctly relate more than two pieces of information (Pe) and use a series of forward computations (Pa) to solve a multistep problem.

**Rationale:** Knowing the current price of region A ($600) allows students to write two proportions to determine the current price of region B \( \left( \frac{3}{2} = \frac{600}{x} \right) \) and region D \( \left( \frac{4}{1} = \frac{600}{x} \right) \). The difference in price between region B ($400) and region D ($150), is $250. Answer choices A and C indicate a correct calculation of one of the region’s prices, but do not represent how much more an advertisement in region B costs compared with region D. Answer choice D represents how much more region A costs than region D, but does not show the difference in price between region B and region D.
2. Naomi needs to buy garments to complete her new soccer uniform, including a pair of socks, two jersey tops, and two pairs of jersey shorts. Naomi’s coach sent home a chart listing the prices of the different items.

<table>
<thead>
<tr>
<th>Parts of Uniform</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Socks</td>
<td>$17.00/pair</td>
</tr>
<tr>
<td>Jersey Tops</td>
<td>$67.50 each</td>
</tr>
<tr>
<td>Shorts</td>
<td>$37.50 each</td>
</tr>
</tbody>
</table>

Naomi’s mother says she will help her pay for it and will contribute $8 for every $2 Naomi contributes. How much money will Naomi need to contribute in order to buy her new uniform?

A $22.70
B $45.40
C $113.50
D $181.60

Key: B

Commentary: This item aligns to 7.RP.A.3 because it involves the application of a proportional relationship to solve a multistep problem.

Multiple Representations: This item involves a series of forward computations (Pa) that requires students to relate several pieces of information (Pe) presented both verbally and in a picture (Pi).

Rationale: This item starts with interpreting the chart and gathering the correct total amount of money required for the soccer uniform ($17 socks, $75 for two pairs of shorts, and $135 for two jersey tops). The proportional relationship stated, 8:2 ($8 dollars of Naomi’s mother’s money to every $2 Naomi contributes), is used to determine the amount Naomi must pay herself, $45.40, of the total $227. Answer choice A shows $\frac{1}{10}$ of the total cost of the uniforms, representing the division of the total cost by the number of combined $8 and $2 contributions made by mother and daughter. Answer choice C incorrectly shows that the two of them have split in half the cost of the new uniform. Answer choice D indicates the amount Naomi’s mother agrees to pay and could be the result of setting up the proportion incorrectly.
4. When David traveled to Mexico he converted $150 U.S. dollars into $1,935 Mexican pesos. David is thinking about purchasing a souvenir that costs $320 pesos, but wants to know how much it will cost him including tax in U.S. dollars. If the sales tax on the souvenir is 15%, what is the total cost of the souvenir in U.S. dollars?

A  $21.08  
B  $24.24  
C  $28.53  
D  $48.00

Key: C

Commentary: This item aligns to 7.RP.A.3 because it involves the use of proportions to solve a multistep problem involving ratios and percents.

Multiple Representations: This item requires non-forward computation (Pb) and the relating of two pieces of information within the problem context (Pe).

Rationale: The sales tax of 15% on $320 pesos yields a total cost of $368 pesos ($1935 \text{ pesos} \div 12.9 \text{ pesos per dollar} \times 1.15$). To determine the total cost in U.S. dollars $368 pesos can be converted into U.S. dollars by dividing $368 pesos by the number of pesos per dollar, $\frac{\$1935 \text{ Pesos}}{150 \text{ U.S. dollars}}$, or 12.9 pesos per dollar. Answer choice A incorrectly multiplies $320 by 85% to determine the price of the souvenir, but correctly converts that amount, $272 pesos, into U.S. dollars. Answer choice B converts $320 pesos into U.S. dollars, but does apply the sales tax on the souvenir. Answer choice D is the result of taking 15% of $320 pesos, or the amount of sales tax on the souvenir in pesos.
10. The star basketball player for a professional basketball team gets hurt $\frac{2}{3}$ of the way through the last basketball season. On the timeline below are the wins and losses of the team with and without their star player.

This year the team expects to play the same number of games as last year, and their star player will only miss the first quarter of the year. If the team maintains the same proportion of wins and losses with and without their star player as they did last year, approximately how many games should fans expect the team to win this year?

A 24  
B 27  
C 36  
D 38  

Key: D  

Commentary: This item aligns to 7.RP.3 and 7.NS.3 because it involves solving a multistep problem that requires the use of proportional relationships and performing operations with rational numbers.

Multiple Representations: This item requires translating between numbers presented in a diagram (Pc) and a description (C3a) in order to predict an outcome (A1) based on a series of forward computations (Pa).

Rationale: This item involves a series of steps that begins with recognizing that the total number of games for both seasons is 60 ($29 + 11 + 7 + 13$). Since it is stated that the team is going to win the same proportion of games as they did the previous year with and without their star player, the ratio of wins to total games played with the player, $\frac{29}{40}$, and the ratio of wins to
total games played without the player, \( \frac{7}{20} \), holds for this year. The team should win \( \frac{7}{20} \) of their first 15 games (5.25 games) and \( \frac{29}{40} \) of the final 45 games (32.625 games), which is 37.875 games or approximately 38 games. Answer choice A is the result of adding \( \frac{1}{4} \) of the 7 games won while the player was injured to \( \frac{3}{4} \) of the 29 games won while the player was not injured. Answer choice B is the result of incorrectly applying the ratio of games won while the player was not injured \( \frac{29}{40} \) to the first quarter of the coming season and applying the ratio of games won while the player was injured \( \frac{7}{20} \) to the final three quarters of the coming season. Answer choice C is the number of games won in the previous season.