AP Calculus Review Worksheet

This packet is a review of the entering objectives for AP Calculus and is due on the first day back to school. It is to be done neatly and on a separate sheet of paper. Have a great summer!

I. Simplifying Rational Expressions

Simplify. (Show your work!)
1. \( \frac{x - 4}{x^2 - 3x - 4} \)
2. \( \frac{x^3 - 8}{x - 2} \)
3. \( \frac{5 - x}{x^2 - 25} \)
4. \( \frac{x^2 - 4x - 32}{x^2 - 16} \)

II. Trigonometric Identities

1. Pythagorean Identities
   - \( \sin^2 x + \cos^2 x = 1 \)
   - \( 1 + \tan^2 x = \sec^2 x \)
   - \( 1 + \cot^2 x = \csc^2 x \)
2. \( \cos 2x = \underline{\text{___________}} \)
3. \( \sin 2x = \underline{\text{___________}} \)

III. Operations with Rational Expressions

1. \( \frac{1}{x + h} - \frac{1}{x} \)
2. \( \frac{2}{x^2} - \frac{10}{x^5} \)
IV. Solving equations

Solve for Z

1. \(4x + 10yz = 0\)

2. \(y^2 + 3yz - 8z - 4x = 0\)

V. Operations with functions

If \(f(x) = \{(3,5), (2,4), (1,7)\}\), \(g(x) = \sqrt{x - 3}\), \(h(x) = \{(3,2), (4,3), (1,6)\}\), \(k(x) = x^2 + 5\) determine the following:

1. \((f+g)(1) = \)
2. \((k-g)(5) = \)
3. \((f \circ h)(3) = \)
4. \((g \circ k)(7) = \)
5. \(f^{-1}(x) = \)
6. \(k^{-1}(x) = \)
7. \(\frac{1}{f(x)} = \)
8. \((kg)(x) = \)

VI. Miscellaneous: Follow the directions for each problem.

1. Evaluate \(\frac{f(x+h) - f(x)}{h}\) and simplify if \(f(x) = x^2 - 2x.\)

2. Expand \((x + y)^3\)
3. Simplify: \( x^2(x + x^2 - x^2) \)
4. Eliminate the parameter and write a rectangular equation for
   \[ x = t^2 + 3 \]
   \[ y = 2t \]

VII. Series

Expand and simplify.
1. \[ \sum_{n=0}^{4} \frac{n^2}{2} \]
2. \[ \sum_{n=1}^{3} \frac{1}{n^3} \]

VIII. Simplifying Expressions

Simplify.
1. \[ \frac{\sqrt{x}}{x} \]
2. \[ e^{\ln 3} \]
3. \[ e^{(1 - \ln x)} \]
4. \[ \ln 1 \]
5. \[ \ln e^7 \]
6. \[ \log_3 \left( \frac{1}{3} \right) \]
7. \[ \log_{\frac{1}{2}} 8 \]
8. \[ \ln \frac{1}{2} \]
9. \[ e^{3 \ln x} \]
10. \[ \frac{4xy^{-2}}{12x} \]
11. \[ 27^{\frac{2}{3}} \]
12. \[ (5a^3)(4a^2) \]
13. \[ (4a^3)^{\frac{3}{2}} \]
14. \[ \frac{3(n+1)!}{5n!} \]

IX. Using the point-slope form \( y - y_1 = m(x - x_1) \), write an equation for the line
1. with a slope of -2, containing the point (3,4)
2. containing the points (1, -3) and (-5, 2)
3. with slope 0, containing the point (4, 2)
4. parallel to 2x - 3y = 7 and passes through (5, 1)
5. perpendicular to the line in problem #1, containing the point (3, 4)

X. Trigonometry
Without a calculator, determine the exact value of each expression.

1. \( \sin 0 \)  
2. \( \sin \frac{\pi}{2} \)  
3. \( \sin \frac{3\pi}{4} \)  
4. \( \cos \pi \)

5. \( \cos \frac{7\pi}{6} \)  
6. \( \cos \frac{\pi}{3} \)  
7. \( \tan \frac{7\pi}{4} \)  
8. \( \tan \frac{\pi}{6} \)

9. \( \tan \frac{2\pi}{3} \)  
10. \( \tan \frac{\pi}{2} \)  
11. \( \cos (\sin^{-1} \frac{1}{2}) \)

12. \( \sin^{-1} (\sin \frac{7\pi}{6}) \)

XI. Domain and Range
For each function, determine its domain and range.

1. \( y = \sqrt{x - 4} \)
2. \( y = \sqrt{x^2 - 4} \)
3. \( y = \sqrt{4 - x^2} \)
4. \( y = \sqrt{x^2 + 4} \)

XII. Determine all points of intersection

1. \( y = x^2 + 3x - 4 \)  
   \( y = 5x + 11 \)
2. \( y = \cos x \)  
   \( y = \sin x \) \quad \text{in the 1st quadrant}
XIII. Solving equations

Solve for $x$, where $x$ is a real number. Show your work.

1. $x^2 + 3x - 4 = 14$
2. $\frac{x^4 - 1}{x^3} = 0$
3. $(x - 5)^2 = 9$
4. $2x^2 + 5x = 8$
5. $(x + 3)(x - 3) > 0$
6. $x^2 - 2x - 15 \leq 0$
7. $12x^2 = 3x$
8. $\sin 2x = \sin x$, $0 \leq x \leq 2\pi$
9. $|x - 3| < 7$
10. $(x + 1)^2(x - 2) + (x + 1)(x - 2)^2 = 0$
11. $27^{2x} = 9^{x - 3}$
12. $\log x + \log(x - 3) = 1$
13. $e^{3x} = 5$
14. $\ln y = 2x - 3$