AP Calculus Review Worksheet

This packet is a review of the entering objectives for AP Calculus and is due on the first day back to school. It is to be done neatly and on a separate sheet of paper. Have a great summer!

I. Simplifying Rational Expressions

Simplify. (Show your work!)

1. \( \frac{x - 4}{x^2 - 3x - 4} \)

2. \( \frac{x^3 - 8}{x - 2} \)

3. \( \frac{5 - x}{x^2 - 25} \)

4. \( \frac{x^2 - 4x - 32}{x^2 - 16} \)

II. Trigonometric Identities

1. Pythagorean Identities

2. \( \cos2x= \)

3. \( \sin2x= \)

III. Operations with Rational Expressions

1. \( \frac{1}{x + h} - \frac{1}{x} \)

2. \( \frac{2}{x^2} \)
3. \[ \frac{1}{3+x} - \frac{1}{3} \]

4. \[ \frac{2x}{x^2 - 6x + 9} - \frac{1}{x+1} - \frac{8}{x^2 - 2x - 3} \]

IV. Solving equations

Solve for \( Z \)

1. \( 4x + 10yz = 0 \)

2. \( y^2 + 3yz - 8z - 4x = 0 \)

V. Operations with functions

If \( f(x) = \{(3,5), (2,4), (1,7)\} \) \( g(x) = \sqrt{x - 3} \) \( h(x) = \{(3,2), (4,3), (1,6)\} \)
\( k(x) = x^2 + 5 \) determine the following:

1. \( (f+g)(1) = \)
2. \( (k-g)(5) = \)
3. \( (f \circ h)(3) = \)
4. \( (g \circ k)(7) = \)
5. \( f^{-1}(x) = \)
6. \( k^{-1}(x) = \)
7. \( \frac{1}{f(x)} \)
8. \( (kg)(x) = \)

VI. Miscellaneous: Follow the directions for each problem.

1. Evaluate \( \frac{f(x + h) - f(x)}{h} \) and simplify if \( f(x) = x^2 - 2x \).

2. Expand \( (x + y)^3 \)
3. Simplify: \[ \frac{2}{3}x^2(x + x^2 - x^3) \]
4. Eliminate the parameter and write a rectangular equation for
   \[ x = t^2 + 3 \]
   \[ y = 2t \]

VII. Series

Expand and simplify.
1. \[ \sum_{n=0}^{4} \frac{n^2}{2} \]
2. \[ \sum_{n=1}^{3} \frac{1}{n^3} \]

VIII. Simplifying Expressions

Simplify.
1. \( \frac{\sqrt{x}}{x} \)
2. \( e^{\ln 3} \)
3. \( e^{(1+\ln x)} \)
4. \( \ln 1 \)
5. \( \ln e^7 \)
6. \( \log_3 \left( \frac{1}{3} \right) \)
7. \( \log_\frac{1}{2} 8 \)
8. \( \ln \frac{1}{2} \)
9. \( e^{3\ln x} \)
10. \( \frac{4xy^{-2}}{12x^\frac{1}{3}y^{-5}} \)
11. \( 27^\frac{2}{3} \)
12. \( (5a^3)(4a^2)^\frac{2}{3} \)
13. \( (4a^3)^\frac{3}{2} \)
14. \( \frac{3(n+1)!}{5n!} \)

IX. Using the point-slope form \( y - y_1 = m(x - x_1) \), write an equation for the line
1. with a slope of -2, containing the point (3,4)
2. containing the points (1,-3) and (-5,2)
3. with slope 0, containing the point (4,2)
4. parallel to 2x-3y=7 and passes through (5,1)
5. perpendicular to the line in problem #1, containing the point (3,4)

X. Trigonometry
Without a calculator, determine the exact value of each expression.

1. sin 0  
2. sin \frac{\pi}{2}  
3. sin \frac{3\pi}{4}  
4. cos \pi  
5. cos \frac{7\pi}{6}  
6. cos \frac{\pi}{3}  
7. tan \frac{7\pi}{4}  
8. tan \frac{\pi}{6}  
9. tan \frac{2\pi}{3}  
10. tan \frac{\pi}{2}  
11. cos(Sin^{-1} \frac{1}{2})  
12. Sin^{-1}(sin \frac{7\pi}{6})

XI. Domain and Range
For each function, determine its domain and range.

1. y = \sqrt{x - 4}
2. y = \sqrt{x^2 - 4}
3. y = \sqrt{4 - x^2}
4. y = \sqrt{x^2 + 4}

XII. Determine all points of intersection

1. \begin{align*} y &= x^2 + 3x - 4 \\
y &= 5x + 11 \end{align*} 
2. \begin{align*} y &= \cos x \\
y &= \sin x \text{ in the 1st quadrant} \end{align*}
XIII. Solving equations

Solve for $x$, where $x$ is a real number. Show your work.

1. $x^2 + 3x - 4 = 14$
2. $\frac{x^4 - 1}{x^3} = 0$
3. $(x - 5)^2 = 9$
4. $2x^2 + 5x = 8$
5. $(x + 3)(x - 3) > 0$
6. $x^2 - 2x - 15 \leq 0$
7. $12x^2 = 3x$
8. $\sin 2x = \sin x$, $0 \leq x \leq 2\pi$
9. $|x - 3| < 7$
10. $(x + 1)^2(x - 2) + (x + 1)(x - 2)^2 = 0$
11. $27^{2x} = 9^{x-3}$
12. $\log x + \log(x - 3) = 1$
13. $e^{3x} = 5$
14. $\ln y = 2x - 3$