Welcome to AP Physics 1!
This assignment is designed to give you an idea of the type of math you will need to use in the course.
We look forward to meeting you in September.

Mr. Byrne and Mr. Gheta

Part 1: Algebra Review

Throughout the year, we'll be rearranging formulas and equations to solve for the variable we want to know. Refresh your algebra skills with the following:

Example:

Suppose we want to know the acceleration, a, in the following formula:

$$y = v_o t + \frac{1}{2} a t^2$$

y is the distance

v_o is the initial velocity

t is the time

Isolate the term with the variable of interest, a. In other words, get the $\frac{1}{2} at^2$ term by itself on one side of the equal sign. To do this we would have to subtract $v_o t$ term from both sides.

$$\begin{aligned} y &= v_o t + \tfrac{1}{2} a t^2 \\ - v_o t &\quad - v_o t \\ \hline y - v_o t &= \tfrac{1}{2} a t^2 \end{aligned}$$

> Common error – dividing the $\frac{1}{2} at^2$ by $\frac{1}{2} t^2$ before isolating the term with the acceleration. If we wanted to do this, we would have to divide <u>all</u> the terms by ½ t^2, not just the $\frac{1}{2} at^2$ term.

Isolate the variable of interest, a. Now we can multiply both sides by 2 and divide both sides by t^2.

$$y - v_o t = \tfrac{1}{2} a t^2$$

$$\frac{2(y - v_o t)}{t^2} = \frac{\cancel{2} (\cancel{\tfrac{1}{2}}\, a \cancel{t^2})}{\cancel{t^2}}$$

$$\frac{2(y - v_o t)}{t^2} = a$$

Example: Solve the following.

$$6 = \frac{18 + 3x}{x}$$

In this case, we have to get all the terms with x's into 1 term. Multiply both sides by x.

$$6x = 18 + 3x$$

Combine the terms with the x's by subtracting 3x from each side

$$\begin{aligned} 6x &= 18 + 3x \\ -3x &\quad - 3x \\ \hline 3x &= 18 \end{aligned}$$

Divide both sides by 3.

$$X = 6$$

> Common error – dividing only the 6 and the 3x terms by x. We can't do this because the entire numerator (18 + 3x) is divided by x in the problem.

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Solve these to refresh your algebra skills.

1. $X + 47 = 95$

2. $55 + a = -78$

3. $1/r = 1/5 + 1/15$

4. $1/32 = 1/f + 1/-8$

5. $37 = \dfrac{314.5}{x}$

6. $5 = \dfrac{3x - 4}{x}$

7. $2x = \dfrac{3x^2 - 16}{x}$

Solve for the given letter

1. $A = p + prt$ for t

2. $A = \dfrac{1}{2} d_1 d_2$ for d_1

3. $f_o = f_s \dfrac{(v + v_o)}{(v - v_s)}$ for v_o

4. $y = mx + b$ for m

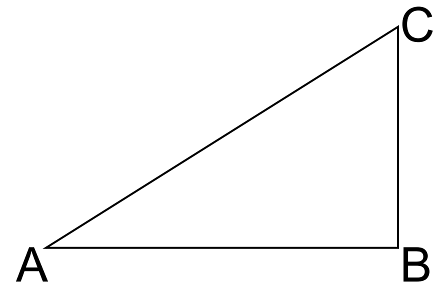
5. $v = \sqrt{\dfrac{GM}{r}}$ for r

6. $F = k \dfrac{Q_1 Q_2}{r^2}$ for r

7. $F = \dfrac{m v^2}{r^2}$ for v

Right Triangles & Trigonometry

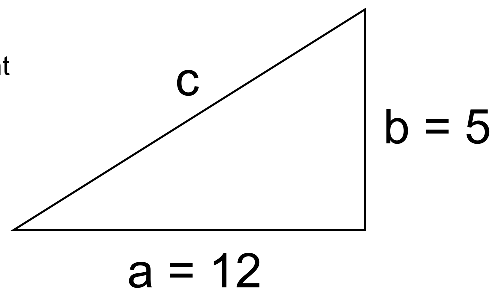
Suppose you wanted to get from point A to point C in the following diagram. You could go directly from A to C, or you could go to the right from A to B and then go straight up from B to C. Thus the direction we go is important. We will define the distance from C to A as our displacement. You'll see shortly that the displacement is a vector. In many cases, we will be interested in the x and y component of a vector. In this case, the x component is AB. The y component is BC.



Pythagorean Theorem

If two of the three sides of a right triangle are known, we can find the 3rd side using the Pythagorean Theorem.

Recall that $a^2 + b^2 = c^2$



$12^2 + 5^2 = c^2$

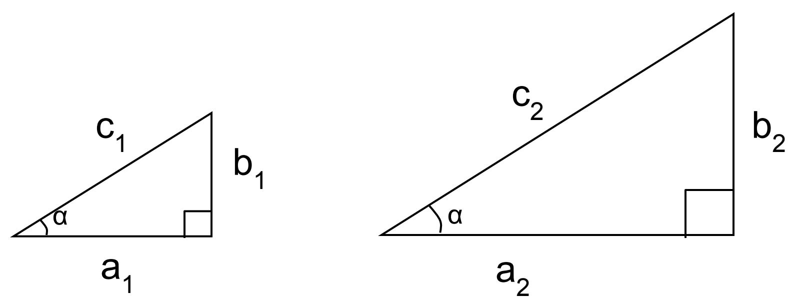
$144 + 25 = c^2$

$169 = c^2$

$13 = c$

Right Triangle Trigonometry

Both of the triangles shown are right triangles. Angle α is the same for both.



Side c is the hypotenuse. Side a is adjacent to angle α. Side b is opposite angle α. If we divided side b by side a for both triangles we would get the same number. The only way we could get a different ratio would be if the angle changed. For example, if the angle increased, side b would have to increase, while side a remained the same. That would cause the ratio to increase. We call the ratio of side b to side a the tangent of the angle. The same logic is true for the ratios of any two sides. We will use three ratios:

Sine → $\text{Sin } \alpha = \dfrac{\text{Opposite}}{\text{Hypotenuse}} = \dfrac{b}{c}$

Cosine → $\text{Cos } \alpha = \dfrac{\text{Adjacent}}{\text{Hypotenuse}} = \dfrac{a}{c}$

Tangent → $\text{Tan } \alpha = \dfrac{\text{Opposite}}{\text{Adjacent}} = \dfrac{b}{a}$

An easy way to remember these is the acronym **SOHCAHTOA** (pronounced so ca toe a)

SOH → **S**ine is **O**pposite / **H**ypotneuse

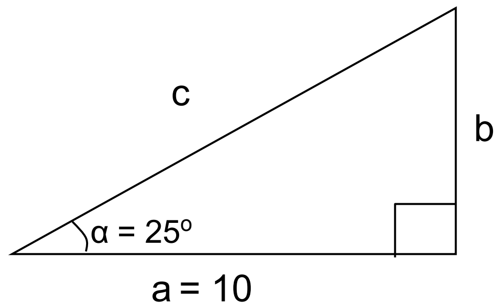
CAH → **C**osine is **A**djacent / **H**ypotenuse

TOA → **T**angent is **O**pposite / **A**djacent

Make sure your calculator is in degree mode, not radian mode.

Examples

Find sides b and c.



$$\text{Tan } \alpha = \frac{\text{Opp}}{\text{Adj}} = \frac{b}{a}$$

$$\text{Tan } 25 = \frac{b}{10}$$

$b = 10 \tan 25 = 4.66$

Now that we know side b, we can use trig or the Pythagorean theorem to find side c.

$$\text{Cos } \alpha = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{a}{c}$$

$$\text{Cos } 25 = \frac{10}{c}$$

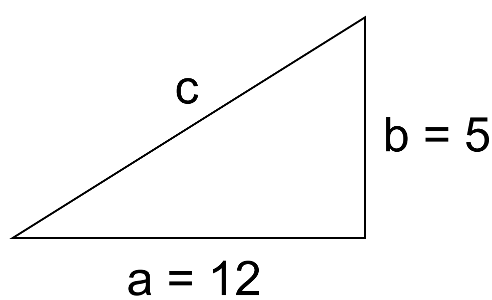
$$c = \frac{10}{\text{Cos } 25} = 11.03$$

Verify this answer by finding c using the Pythagorean theorem.

Finding angles

Since each angle has a unique sine, cosine and tangent value, we can use these values to find the angle. We call these functions the inverse tangent, inverse sine, or inverse cosine.

$$\alpha = \tan^{-1} \frac{\text{Opposite}}{\text{Adjacent}} = \tan^{-1} \frac{b}{a}$$



We'd read this equation as

"α is the angle whose tangent is b/a"

For our example,

$$\alpha = \tan^{-1} \frac{\text{Opposite}}{\text{Adjacent}} = \tan^{-1} \frac{5}{12}$$

The inverse functions are normally above (shift or 2nd function) the standard trig function button on your calculator.

$\alpha = 22.62^{\circ}$

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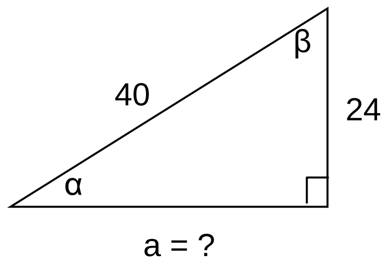


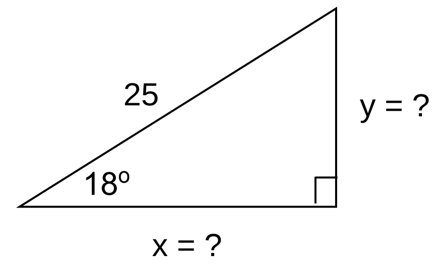
1. What is the length of side a?

3. What is the sine of angle α ?

4. What is the cosine of angle α ?

5. Angle α = ______________degrees.

6. Angle β = ______________degrees. (use trig and check to see if all the angles add to 180°)



1. What is the length of side x?

2. What is the length of side y?

3. Use the Pythagorean theorem to check your answers.

If the 25 was a vector quantity (see chapter 1), the values you found for x and y would be called the x and y components.

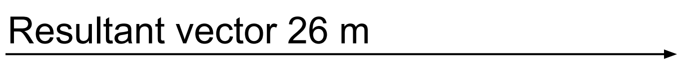
Part II: Vector Addition and Subtraction

Vectors have both magnitude and direction, thus we must take the direction into account when adding or subtracting vectors.

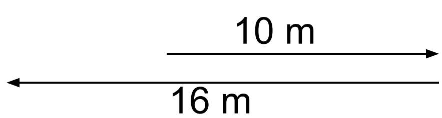
Adding Vectors:

1. The simplest case occurs when vectors are collinear. Vectors are normally represented by arrows. When adding, we used a tail-to-head relationship. Thus the tail of the vector being added to the original vector is placed at the head of the original vector. If they are in the same direction, simply add the magnitudes of the vectors. The direction will be the same as that of the individual vectors. For example, if someone walked 10 m East and then 16 meters East, we would draw the vectors as:

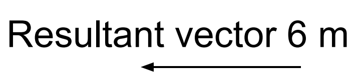
The resultant vector would be 26 m East. It is drawn from the tail of the first vector to the head of the second one.



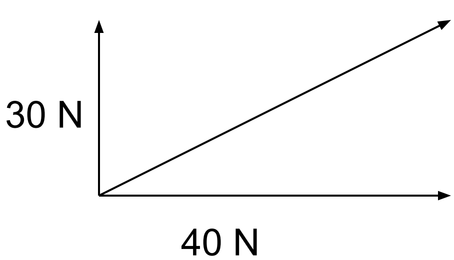
If vectors are in the opposite direction, add them, keeping in mind they have opposite signs.



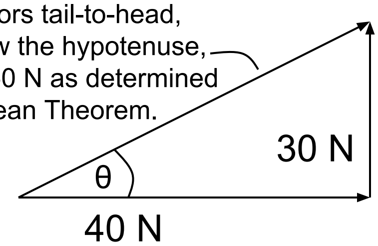
The resultant vector would be 6 m West.



2. The next simplest case occurs when vectors are perpendicular to each other.. We use the Pythagorean theorem to add these vectors. For example, if we had a force of 40 N pushing an object due West and a force 30 N pushing an object due North, we know the object would move along a path between the two forces as shown by the dashed line.



Arranging the vectors tail-to-head, the resultant is now the hypotenuse, so it would equal 50 N as determined from the Pythagorean Theorem.



To find the angle, θ, that the resultant force acts along, we can use the inverse tangent function.

$\theta = \text{Tan}^{-1}(30/40) = 37^{o}$ above the horizontal

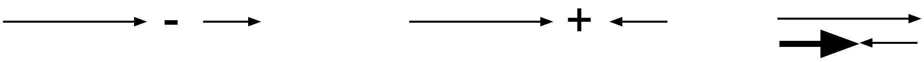
This tells us that the two original forces could be replaced by a single force of 50N acting at 37^{o} above the horizontal. The 50N force at 37^{o} is the sum of the original forces.

Subtracting Vectors:

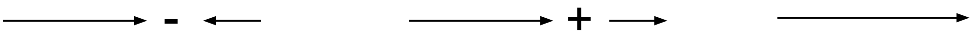
To subtract two vectors, we simply have to take the opposite of the second vector and add it to the first.

Some examples:

30 m E – 10 m E = 30 m E + 10m W = 20 m E



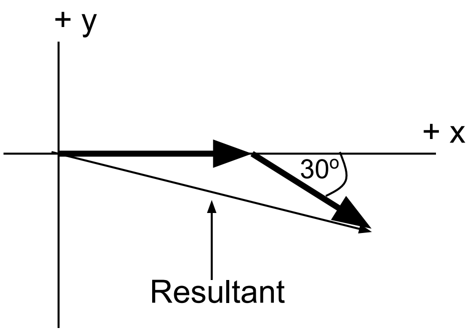
30m E – 10 m W 30m E + 10m E = 40 m W



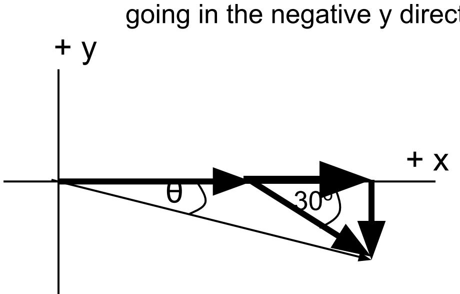
Adding Vectors that are not collinear and not perpendicular:

To add vectors such as 70 m due E to 50 m at 30° S of E, we need to break the vectors into x-y components.

1. Start by drawing a picture. Don't skip this step; it will help you avoid direction errors.
2. Set up an x-y system. Label the positive and negative directions.



3. Use trig to determine the x and y components of the vector. Be sure to take directions into account by using + and – signs.
   1. The 70 meter vector lies along the +x axis. Its x component is its full length, 70m and it has no y component.
   2. The 50 meter vector must be broken into components. Draw the components in the x and y direction to make a right triangle. The original vector is the hypotenuse.
   3. The x component of the 50 m vector can be found using the cosine function in this case. The y component can be found using the sine function Note the negative sign since we are going in the negative y direction.



Cos 30 = x/50, so x = 50 cos30 = 43.3 m

Sin 30 = -y/50, so y = -50 sin30 = -25 m

> Note: the x component won't always be cosine. It depends on the angle you use.

4. Make a table to organize your data.

Vector	x	y
70 m	70	0
50 m	43.3	-25
Resultant	113.3	-25

5. Add the x components and the y components.
6. Use the Pythagorean Theorem to determine the resultant vector.

$$\sqrt{113.3^2 + 25^2} = 116 \text{ m}$$

7. Use the $\tan^{-1}$ function to find the angle

$\theta = \tan^{-1}(25/113.3) = 12.4^\circ$

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1. Add these vectors:

15 m/s N + 20 m/s S

40 m E + 60 m N

2. Subtract

28 m N – 15 m S

3. Add:

22 m 15° N of E + 64 m 25° W of N + 38m due N (Draw the picture first)