Lessons - Quadratic Equations

COURSE: ALGEBRA
Unit 6: Quadratic Equations
Time Frame for developmental lessons: 20 days
Time Frame for Charter Schools: 20 days

Unit Resources:

Common Core Standards for this Unit

Sample Lesson Sequencing

Unit Chunks:

1. Quantitative Relationships
2. Solving Quadratic Representations
3. Analyzing Quadratic Representations

Essential Qs	Core Math	Unit Resources
How can change be measured?	**Quantitative Relationships for Quadratics** Students will analyze quantitative relationships for quadratic expressions and understand how to recognize the relationship between the quantities and powers in factored form and standard form of a quadratic. Students will: ● Create quadratic and/or exponential equations/expressions by appropriately assigning variables and numbers to quantities in a situation and can explain the meaning of each value/term used ○ understands how to use variables to represent an unknown/changing quantity or a constant to represent a known/unchanging value ○ compare quadratic and exponential functions: exponents can	**Writing quadratic rules, multiplying binomials, and investigating arithmetic properties of quadratic expressions and equations:** ● Crisscross Numbers (this task is revisited from Unit 3, students might revisit number properties or use binomials and carefully consider relationship between quantities within the situation) ● Calendar Patterns (revisited from Unit 3, heavy emphasis on justifying and proving truth value

	be variables too (not just numbers) ○ *Students will understand what each part of the equation represents* ● investigate the relationship between the quantities and powers in factored form and standard form of a quadratic ○ convert quadratic expressions from factored form to standard form ○ visually represent the relationship between factored form and standard form and can justify the sum of areas for its component parts ● use the distributive property to multiply a quadratic expression in factored form to standard form ○ use the distributive property to justify why two representations are equivalent ● apply patterns of quadratic expressions to multiply polynomials with any number of terms or degrees/powers and justify approach ● describe the exponential property of multiplication between two variables ● investigate and describe the relationship between quantities in standard form of any polynomial and its factored form ○ understands the relationship between the degree of an expression in standard form and the product of the leading coefficients of each factor ○ recognize special cases: perfect squares, difference of squares, leading coefficient other than 1 ● use the structure of an expression to identify ways of rewriting it and can justify reasoning ○ factor perfect square trinomials with any even degree (2, 4, 6, etc.) ○ factor difference of squares with any even degree ○ factor out the greatest common monomial factor for any polynomial ○ recognize and justify when a polynomial cannot be factored *SWBAT describe work by using these terms appropriately and consistently:* distributive property, degree/power, factor, perfect square, difference of squares, perfect square trinomial, expression, equation, factored form, standard form, exponent, equivalent, polynomial, coefficient, leading coefficient	of a rule which may involve multiplying binomials) ● Number Patterns (easy entry task involving consecutive number patterns and perfect squares, helps students evaluate relationships between quantities in a quadratic situation) ● Rectiles (can be used with Algebra tiles, helps build foundations for completing the square in Chunk 2 of this unit) ● Dots & Squares (question #4 good transition task to Chunk 2 about solving equations) **Analyze relationship between number quantities (in support of factoring):** ● Pedro's Tables (2008) **Evaluating equivalency for quadratic or exponential expressions:** ● Seeing Dots (consider equivalency for quadratic representations, Illustrative Mathematics) A-SSE.3a,b ● http://www.illustrativemathematics.org/illustrations/1305 **Other Resources and support on how to use them**

back to top

Essential Qs	Core Math	Unit Resources
What does a solution represent?	**Solving Quadratic Representations** Students will understand solving equations as a process of reasoning and explaining the reasoning. Students will: ● recognize and explain the significance of quadratic roots in the context of a situation or an abstract equation ○ describe the meaning of roots in context of a situation/real world problem ● determine the roots of a quadratic equation in factored or standard form ● utilize the zero product property to solve for roots with factors and can mathematically justify this approach ○ factoring a quadratic equation highlights the roots of the function because of the property of multiplication in which when ab = 0 either a = 0 or b = 0 ○ algebraically solve for the roots in standard form by factoring ● determine the roots of a quadratic equation in vertex form and can identify the distance between the roots and the vertex ○ algebraically solve for the roots in vertex form using inverse operations and justify each step ○ examine case where vertex is the x intercept ● algebraically determine the roots of a quadratic equation by completing the square ○ recognize the connection between vertex form and completing the square ● utilize the quadratic formula to solve for roots of a quadratic function and justify reasons for needing/using it ○ describe how the quadratic formula is related to patterns for completing the square ○ recognize how the quadratic formula relates to completing the square ○ complete the square for a quadratic equation in standard form to derive the quadratic formula ● Manipulate equations to identify/highlight specific features about the	**Make sense of and solve quadratic equations:** ● Graphs (compares linear and quadratic graphs, review of content from units 2-4, question #3 students may factor to solve) ● Number Towers (extend quadratic patterns, describe relationship between quantities, and solve quadratic equation via factoring or another algebraic method) ● Patchwork (extend quadratic patterns, describe relationship between quantities, and solve quadratic equation via factoring or another algebraic method) **Evaluate relationships between quantities to build factoring fluency:** ● Katie's Pattern ● Sum & Product (compare the relationships between sum and product of two numbers that helps students factor) ● Factoring Trinomials (FAL, cart sort matching activity) **Understanding perfect squares and building foundations for completing the square:**

	quadratic function and the language to communicate their reasoning ○ factoring a quadratic equation highlights the roots of the function because of the property of multiplication in which when ab = 0 either a = 0 or b = 0. ○ completing the square highlights the vertex, axis of symmetry, and the limitations on the range of the parabola ● describe and quantify the roots of a quadratic equation using the discriminant ○ analyze the quadratic formula and recognize patterns that cause the quadratic to have 0, 1, or 2 real roots ○ recognizes when the parabola with not have any real roots, understands how the discriminant can be used to help determine this (and justify this method through its connection with completing the square) *SWBAT describe work by using these terms appropriately and consistently:* solution, solution set, factor, zero product property, roots, zeros, x intercept, unfactorable, square root, axis of symmetry, perfect square, vertex, completing the square, quadratic formula, real, non-real, discriminant, factored form, expression, equation, term, coefficient, leading coefficient, constant, mono/bi/poly-nomial, minimum, maximum	● Numbers **Other Resources and support on how to use them**

back to top

Essential Qs	Core Math	Unit Resources
How do we determine the best method for obtaining a solution?	**Equivalent Representations** Students will understand that equations can be represented in a variety of equivalent forms with identical solutions. Students will understand that the solutions can be verified by using different methods, and if done correctly, the	**Use several methods to verify solutions for a quadratic function:** ● Breaking Distance (use multiple methods to verify a solution

	different methods should produce the same result. Students will: ● Make comparisons between equivalent expressions ○ can identify identical equivalent features of a parabola given two different forms of its quadratic equation to prove that the algebraic representations are equivalent ○ uses order of operations and other number properties (distributive property, etc.) to prove that two expressions/equations are equivalent ○ uses a variety of methods (concrete and abstract) to prove equivalency, for example substitutes a specific value and simplifies ● Compare equivalent algebraic equations that model the same situation ○ explain how each equation describes/interprets the situation ○ arithmetically prove that two equations are equivalent ○ understands how different equations can model the same situation, there are many different ways to write quadratic functions that describe the same quantity ● Construct viable arguments to justify a solution method articulating assumptions, which includes the assumption that the original equation has a solution, Quadratic equations should be solved using different methods in order reveal different properties of the function ○ analyze features of a quadratic equation to determine and justify which method (factoring, completing the square, quadratic formula, graph) is the most efficient ○ understands the relationship between the quadratic formula and completing the square, derive the quadratic formula by attempting to complete the square of a quadratic equation in standard form ● Make sense of equivalent expressions/equations in order to extend reasoning to other models/expressions/equations, flexibly understands how numbers and values are organized within an algebraic expression or equation and can create equivalent models. ● Extend their understanding of equivalent quadratic expressions to other polynomial expressions and be able to manipulate these expressions using arithmetic on the polynomials. ● Understand what their solutions mean within different contexts. ○ Students will understand how to manipulate equations in order to solve for a specific quantity	including graphing and the quadratic formula, Illustrative Mathematics) A-REI.4b ● Spring Board Dive (use variety of methods to evaluate key parts of a quadratic function in the context of a situation) ● Quadratic Puzzles (online tool or can be printed) ● Two Squares are equal (Illustrative Mathematics) A-REI.4b, A-REI.11 **Other Resources and support on how to use them**

	○ accept and reject possible solutions based on the context of the situation and the limitations on reasonable domain and range values ● Articulate the relationship between an algebraic representation with the situation (words, table, visual pattern) that it represents ○ determine if an equation/expression accurately portrays a given scenario in words, can justify if the words and algebraic model align ○ determine which types of equations can be used to model different situations, specifically differentiating exponential vs. quadratic *SWBAT describe work by using these terms appropriately and consistently*: equivalent, domain, range, solution, no solution, solution set, factor, zero product property, distributive property, roots, zeros, x intercept, unfactorable, square root, axis of symmetry, perfect square, vertex, minimum, maximum, completing the square, quadratic formula, real, non-real, discriminant, standard form, vertex form, factored form	

back to top

Common Core Learning Standards:

Use properties of rational and irrational numbers

- **N-RN.B.3** Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Perform arithmetic operations on polynomials.

- **A-APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Understand the relationship between zeros and factors of polynomials.

- **A-APR.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. *Tasks are limited to quadratic and cubic polynomials in which linear and quadratic factors are available.*

Interpret the structure of expressions

- **A-SSE 1** Interpret expressions that represent a quantity in terms of its context.
- **A-SSE 1a** Interpret parts of an expression, such as terms, factors, and coefficients

- **A-SSE 1b** Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ as the product of *P* and a factor not depending on *P*.
- **A-SSE 2** Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.

Write expressions in equivalent forms to solve problems

- **A-SSE 3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
- **A-SSE 3a** Factor a quadratic expression to reveal the zeros of the function it defines.
- **A-SSE 3b** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- **A-SSE 3c** Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15t can be rewritten as (1.15 1/12)12t ≈ 1.01212t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

Analyze functions using different representations.

- **F-IF 8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- **F-IF 8a** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Create equations that describe numbers or relationships.

- **A-CED 1** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from ~~linear and~~ quadratic functions~~, and simple rational and exponential functions.~~*

Solve equations and inequalities in one variable

- **A-REI 4** Solve quadratic equations in one variable.
- **A-REI 4a** Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
- **A-REI 4b** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. **Recognize** when the quadratic formula gives complex solutions ~~and write them as *a* ± *bi* for real numbers *a* and *b*~~
- **A-REI.11** Explain why the x-coordinates of the points where the graphs of the equations y=f(x) and y=g(x) intersect are the solutions of the equation f(x)=g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, ~~exponential, and logarithmic functions~~.
- **A-APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- **A-APR.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

back to top