Lessons - Modeling with Functions

COURSE: ALGEBRA
Unit 1: Modeling with Functions
Time Frame for developmental lessons: 20 days
Time Frame for Charter Schools: 18 days

Unit Resources:

Common Core Standards for this Unit

Sample Lesson Target Sequencing

Unit Chunks:

1. Function relationships
2. Multiple Representation
3. Rate of Change
4. Families of Functions

Ongoing: Unit Conversions

Essential Questions	Core Math	Unit Resources
How can we model real life situations?	**Function relationships** Students will understand how functions relate pairs of values and how functional relationships are different from other relationships. (f*ocus on graphical representations*). • functions provide a tool for describing how variables change together and a means for describing and understanding **relationships between** variables[1], tells the story about how two quantities are related	**You could start each day with:** • Graphing Stories • Visual patterns (just have students draw another patterns in the sequence- creating equations is introduced in Unit 2) • Dot Talks (or other types of extending visual patterns) • SVMI Math Talks (various

[1] FROM NCTM Developing Essential Understanding of Expressions, Equations & Functions 6-8

	• determine if a relation is a function by identifying functions and non functions in graphs, tables, and sets of points • One value (input) of a function determines the other value (output), given an input you can determine an output, describe connections between context and algebraic representations which use function notation • functions are used to relate real world quantities and describe patterns, graphs contain specific features that let us interpret a real life situation, describe the restrictions on the domain of all functions based on real world context • identify and relate the domain of a function to its graph and why a domain is appropriate for a given situation (example: $ per person is only positive integers), identify key features of a graph (rate, intercepts, maximum and minimums, quadrant, end behavior) and interpret these in the context of the given quantities *SWBAT describe work by using these terms appropriately and consistently:* domain, range, dependent, independent, function notation, vertical line test, f(x), continuous, discrete, relation, function	numeracy talks, etc.) **Identifying Variables:** • How do we use variables? **Understanding the relationship between two variables:** • Dan Meyer blog • Walk the Line • A Swimming Race • Number Pairs • The Number Cruncher (especially good for recognizing and describing patterns between inputs and outputs of a function) • European Trip (#2 utilizes relationship via a graph and #3 wants an explanation of the relationship) • Lattice Fence (students will think deeply about the relationship between variables and how one variable relates to another in the context of rate of change,uses some geometry vocab that would need to framed) **Interpreting the features of a graph in the context of a situation:** • Buses • Vincent's Graphs (clarify what "stone" is in #3, ask students to explain the graph they make in #3 for more insight into student thinking)

back to top

Essential Questions	Core Math	Unit Resources
How can we use multiple representations to make sense of the world?	**Multiple Representations** The function relationship can be represented in multiple ways. ● functions can be represented in multiple ways and these representations are useful in analyzing patterns of change[1] ● some representations of a function may be more useful than others depending on how they are used[1], different representations of functions can highlight different information about the situation that it models ● make fluent connections between different representations including graphs, tables, equations/rules, and situations (developed through inspection), identify the domain and range of a function from multiple representations ○ write a given domain or range as an interval or using set builder notation ○ identify functions and non functions from multiple representations including graphs, tables, and sets of points ● Translate from algebraic representations to graphic or numeric representations and identify key features using the various representations *SWBAT describe work by using these terms appropriately and consistently*: domain, range	**Connecting different representations to the same function:** ● Interpreting Distance-Time Graphs Formative Assessment Lesson (FAL) (matching situations, graphs, and tables and interpreting key features of a graph) ● Represent'n **Patterns of change between two variables can be represented in multiple ways:** ● Shelves **Other Performance Tasks:** Use these tasks as needed

back to top

Essential Questions	Core Math	Unit Resources
How do we measure change?	**Rate of Change** A rate of change describes how one variable quantity changes with respect to another; in other words, the rate of change describes the covariation between two variables[1].	**Using Situations to think about different rates of change:** ● Crystal Earings ● Dan Meyer blog ● Walk the Line

	• One important way of describing functions is by identifying the rate at which the variables change together[1] • students will know that rate relates two quantities • calculate rates of change from multiple representations of functions including verbal descriptions and graphs • know the difference between constant rates of change and non-linear rates of change and be able to explain why a line results from a constant rate of change • linear functions arise in situations where the rate of change between two quantities is constant, Quadratic functions are characterized by rates of change that change at a constant rate, In exponential growth, the rate of change increases over the domain; in exponential decay, the rate of change decreases over the domain (introductory and descriptive only)[1] *SWBAT describe work by using these terms appropriately and consistently:* average rate of change, instantaneous rate of change, covariation, increasing, decreasing	• Graph game **Analyzing Rates of Change by reading a graph:** • A Cross Country Run (note #7 deals with average speed which is explicitly introduced in Unit 2) • Bike Ride (*note: this is not the same as the Bike Ride that is listed as the Unit 1 Final Task)* • Going to Town (#4 also ties unit conversions in nicely)

back to top

Essential Questions	Core Math	Unit Resources
How can we model real life situations?	**Families of functions** Families of functions help classify different types of function relationships and help model different kinds of real world phenomena. • know the difference between constant rates of change and non-linear rates of change and be able to explain why a line results from a constant rate of change • linear functions arise in situations where the rate of change between two quantities is constant, Quadratic functions are characterized by rates of change that change at a constant rate, In exponential growth, the rate of change increases over the domain; in exponential decay, the rate of change decreases over the domain (introductory and descriptive only)[1]	**Comparing Linear & Quadratic Functions:** • Comparing Patterns (use rate and change and multiple representations to analyze distinguishing traits of linear and quadratic function families) **Describe general features of different families:** • Understanding Graphs (match graphs with equations, descriptions about the relationship

	• relate the concept of domain to each function family studied • compare the average rate of change for a linear function with a quadratic function over different intervals *SWBAT describe work by using these terms appropriately and consistently:* linear function, quadratic function, exponential function, step function, domain	between variables, and situations, contains a rational function that students will also encounter in the FAL) **Other Performance Tasks:** Use these tasks as needed

back to top

Essential Questions	Core Math	Unit Resources
How do units and relationships between units help us to understand problems?	**Unit Conversions** Unit conversions should take place during the course of the entire unit as appropriate. Students will know that a quantity is a number with an attached unit and will understand how to convert and relate (products and quotients) between different quantities • students understand that a unit conversion is a unit rate that is linear and proportional and if graphed is linear • select appropriate units and round appropriately • use dimensional analysis and/or proportions to convert from one unit to another • SWBAT choose and create an appropriate scale in a graph • identify key features of a graph (rate, intercepts, maximum and minimums, quadrant, end behavior) and interpret these in the context of the given quantities	**Preliminary Support:** • A Million Dollars **Making sense of a situation using conversions:** • Leaky Faucet • Olympic Event • Speeding Ticket (#3 asks students to write an expression which is an extension into Unit 2, other questions focus on calculating rates and considering units to make sense of solutions) **Conversion relationship between two variables and its graph:** • Bell Peppers **Extending patterns with conversions:** • Lattice Fence (see also the reference to this task in *Function Relationships* section)

back to top

Common Core Learning Standards:

Reason quantitatively and use units to solve problems.

- **N-Q 1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
- **N-Q 2** Define appropriate quantities for the purpose of descriptive modeling.
- **N-Q 3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *The greatest precision for a result is only at the level of the least precise data point (example: if units are tenths and hundredths, then the appropriate level of precision is tenths).*

Understand the concept of a function and use function notation

- **F-IF 1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x)*.
- **F-IF 2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Interpret functions that arise in applications in terms of the context

- **F-IF 4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
- **F-IF 5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

Analyze functions using different representations

- **F-IF 9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Construct and compare linear, quadratic, and exponential models and solve problems

- **F-LE 1** Distinguish between situations that can be modeled with linear functions and with exponential functions.

- **F LE 1a** Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- **F LE 1b** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- **F LE 3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Interpret expressions for functions in terms of the situation they model

- **F LE 5** Interpret the parameters in a linear or exponential function in terms of a context.

back to top