Lessons - Linear & Exponential Functions

COURSE: ALGEBRA
Unit 2: Linear & Exponential Functions
Time Frame for developmental lessons: 15 days
Time Frame for Charter Schools: 19 days

Unit Resources:

Common Core Standards for this Unit

Sample Lesson Sequencing

Unit Chunks:

1. Interpret Function Features
2. Analyze Multiple Representations
3. Creating and Using Formulas/Rules

Essential Questions	Core Math	Unit Resources
How can we measure change?	**Interpret Function Features** Students will be able to interpret key features of a function through various representations (*focus on the rate of change and initial value*). ● Understand that **functions can be used to predict future states/generate next steps,** determine the next values in a pattern, and describe how two quantities are related and change together. Students will be able to articulate the patterns using words, **recursive forms** and **explicit forms** ○ use the rate of change to continue the pattern within a function to determine an output given an input and an input given an output ● Articulate the meaning of rate of change and initial value (without using	**You could start each day with:** ● Get the Count Out **Using rate of change to extend a pattern:** ● Festival Lights (good extension of rate of change from Unit, more quantitative focus between in and outputs) ● Make Half (work with fractions and solve a puzzle by identifying the pattern/rate of change and extending it)

	the terms "slope", "y-intercept", or - if in context - the variables "y" or "x.") ○ explain slope as graphic representation of the rate of change between dependent and independent variables; in models, rate of change is more than just an abstract number- it has units and it's measuring something per something (for example, miles per hour) ○ explain y-intercept as the graphic representation of initial value, when the input (x-coordinate) equals zero ● Compare any two functions using rate of change and/or initial value (**F IF 6**) ○ Members of a family of functions share the same type of rate of change. ○ sort linear, exponential and other functions, distinguish between situations that can be modeled with linear functions and with exponential functions (determine when each function may be of use). ○ estimate the rate of change from a graph ○ identify y-intercept from graph & initial value from a table (integer and rational values) precisely if possible (including use of graphing calculator) and approximately if not (ex circle the y-intercept on an unlabeled graph) ○ Calculate and interpret the average rate of change for any representation of any function (over a specific interval) ○ consider and describe shifting parameters: what happens when the rate of change and/or y-intercept changes? ● Recognize situations in which one quantity changes at a constant rate per unit interval relative to another (**F-LE 1b**) ○ Linear relationships (covariation) are characterized by constant rate of change. The constancy of a linear function's rate of change is one of the main characteristics that determines the kinds of real world phenomena that it can model. ○ Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals (**F-LE 1a**) *SWBAT describe work by using these terms appropriately and consistently:* function, independent, dependent, rate of change, domain, range, interval, average rate of change, instantaneous rate of change, covariation, initial value, y intercept, x intercept, constant	**Creating Graphs & Interpreting Quantitative Features:** ● Journey (good bridge task between Unit 1 and 2 content) ● *Project:* Students create a graph that models a youtube video ● Revisit Graphing Stories **Calculate the rate of change and interpret it from a graph (F IF 6):** ● The Cog Railway ● Worksheet: Rate of Change from graphs ●

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Essential Questions	Core Math	Unit Resources

How do we quantify the differences between representations?	**Analyze Multiple Representations**
	
	Changing the way that a function is represented (e.g. algebraically, with a graph, in words, or with a table) does not change the function, although different representations highlight different characteristics.
	
	● identify important features/information about a function through a variety of representations and can
	  ○ identify where the rate of change and initial value show up in all representations of linear functions (and non-linear as well!)
	● for a single function, make sense of/reason how various representations align with one another
	  ○ transform functions from one representation to another, translate between an equation, a graph, a bunch of words, and a table of values
	  ○ investigate connections between multiple representations
	  ○ match a function's table to its graph
	  ○ graph lines from slope-intercept equations, point-slope equations, and standard-form equations, from tables, and from stories
	● for a single function, articulate what each representation highlights about the function, different representations of functions can highlight different information about the situation that it models (**F IF 7, F IF 7a, F IF 9**)
	  ○ understand the pros and cons of each representation (ie a table of values can't conclusively define a certain type of function, while a graph can't pinpoint intercepts with certainty)
	● compare and articulate the properties of two functions each represented a different way (focus on comparing linear & other families of functions represented in different ways)
		Recognizing the relationship between a situation, its graph and an equation:
		● Driving (#4 compare two graphs using rate of change)
		● Vacations (#3 describe the relationship between quantities in an equation)
		
		Matching equations and graphs:
		● Linear Graphs
		● Graphs
		
		Writing Equations that match situations:
		● Words & Equations
		● It is Just an Expression (focuses on understanding the meaning of each variable and value within an equation and how the equation models a situation, good extension of making sense of variables from Unit 1)
		
		Recognizing connections between a graph, table of values, and equation:
		● Use a Noticing/Wondering protocol: Comparing Different Forms of the Equation of a Line

	SWBAT describe work by using these terms appropriately and consistently: rate of change, y intercept, x intercept, constant, linear, exponential, quadratic, domain, range	

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Essential Questions	Core Math	Unit Resources
How do we quantify the relationships between quantities?	**Creating & Using Formulas/Rules** Function rules (equations) can be generated to describe relationships between two quantities, usually *x* and *f*(*x*), where *f*(*x*) is some output value that depends on the input value *x* **(F-BF 1, F-BF 1a).** Students will: ● Understand that linear functions have formulas of the form f(x) = mx + b which can be examined over intervals (often in real world contexts) or over the coordinate plane ● Recognize and appreciate that equations can be used to create other representations (and vice versa). Students will also: ○ sketch the graph of any equation; ○ determine if two lines are parallel, given their equations in any form; ○ transform any graph, table of values (two points), or description into a mathematical equation that describes the function; ○ compare the features of an equation with other representations, for example: a function *f*(*x*) that has a *y*-intercept of 4 would need to have an equation such that *f*(0) = 4. Similarly, a table of values for this function would be expected to have the point (0, 4); ○ investigate and generalize how changing the coefficients of a function affects its graph, and play with parameters (ie what happens when you go by 3s? start at 6? *make a new equation by adding 1? write a linear equation steeper than…)*.	**Creating equations to model sequences and patterns:** ● Visual Patterns (these patterns model all kinds of function families so make sure the pattern chosen aligns with the unit) ● Game: Guess my rule ● Necklaces ● Parallels (this task involves some geometry vocab but the focus is on extending visual patterns, creating an equation to model the pattern and using it to extend the pattern, great distinction between quadratic and linear rate of change) ● Courthouse Steps (contains multiple linear patterns, create an equation from one of the linear patterns and compare the patterns to predict which linear function could contain a particular output, useful transition to Unit 3 content) **Creating and analyzing equations using rate of change:**

	• Generate equations given features of its graph and explain how the graph of an equation visually represents the solution set.
	  ○ determine whether a given point is on a line, given the equation of the line
	  ○ write the equation of a line, given its slope and the coordinates of a point on the line
	  ○ write the equation of a line parallel to the x- or y-axis
	• Understand the differences between linear functions and exponential functions and when each function may be of use (**F-LE 1 & 2**). Students will also:
	  ○ understand that in simplest terms, a linear function is one that takes the form $y = mx + b$ and an exponential function is one in which $y = a^x$;
	  ○ recognize differences between the rates of change of linear and exponential functions, (through graphs, tables, or other media);
	  ○ accurately graph an exponential equation by hand and using technology.
	  ○ analyze arithmetic and geometric sequences (via a graph, situation, or table/pairs of values)
	
	SWBAT describe work by using these terms appropriately and consistently:
	rate of change, linear function, exponential function, parallel, input, output
		• Cups (#7 deeper analysis into the impact of rate of change on the outputs of a function)
		• Apartment Numbers (#5 extends to Unit 3 but it is possible to answer using patterns)
		• Boxes of Chocolates (#3 find an output given an input into the function, #5 determine an output by extending the pattern)
		
		Utilize graphs to make sense of situations and create equations:
		• Circles & Squares (8th grade task, students extend linear patterns, make sense of linear function on a graph and use it to identify appropriate, corresponding equation)
		• Truffles
		• David Wees' engineering task
		
		Creating and analyzing exponential functions from a situation:
		• Dan Meyer's 3 Act: Double Sunglasses (exponential decay)
		• Dan Meyer's 3 Act: Fry's Bank (exponential growth)

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Common Core Learning Standards:

Understand the concept of a function and use function notation

- **F-IF 3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n–1) for n ≥ 1.*

Interpret functions that arise in applications in terms of the context

- **F IF 6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations

- **F IF 7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- **F IF 7a** Graph linear functions and show intercepts, maxima, and minima.
- **F IF 7b** Graph ~~square root, cube root,~~ and piecewise-defined functions, including step functions and absolute value functions.
- **F IF 9 (only linear)** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Build a function that models a relationship between two quantities

- **F-BF 1** Write a function that describes a relationship between two quantities.
- **F-BF 1a** Determine an explicit expression, a recursive process, or steps for calculation from a context.

Construct and compare linear, quadratic, and exponential models and solve problems

- **F-LE 1** Distinguish between situations that can be modeled with linear functions and with exponential functions.
- **F-LE 1a** Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- **F-LE 1b** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. (only linear)
- **F-LE 2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- **F LE 5** Interpret the parameters in a linear or exponential function in terms of a context.

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