Name___________________________ AP Calc BC

Summer Packet

This packet is a list of topics you should be able to do by the time you get into the fall semester of AP Calculus BC. These are all examples of the topics that you should *know* and *understand*. It is not a list of questions that I will ask you. You should be able to do almost all the material in this review packet ***without a calculator***. I know, that's going to be tough for some but I have faith in you! If you have questions please feel free to e-mail me at trizzotti@klschools.org. BTW- There will be a pre-requisite test at the beginning of the year on all of these topics.

The topics are broken up into two parts:

Part 1: PreCalculus/Algebra topics

Part 2: Calculus topics

Part 1 is due to me **by August 1st**. It can be dropped off in my mailbox at the school or it can be scanned and e-mailed to me. The promptness of handing this in will be part of your pre-requisite test grade. It will be checked but not graded on accuracy.

Part 2 will be due **the first full day of classes**. It will be checked for completeness, not accuracy. This will also be part of your pre-requisite test grade.

Part 1:

1. Explain why the following expressions are equal

$$\sqrt[3]{8^4} = \sqrt[3]{8}^{\,4} = 16$$

2. Factor each expression completely
   a. $x^3 - 27$

   b. $8x^3 + 125$

3. Answer each of the following questions about logs and exponentials:
   a. What is the domain of $y = \ln x$?
   b. What is the range of $y = \ln x$?
   c. What is the y-intercept of $y = \ln x$?
   d. What is the x-intercept of $y = \ln x$?
   e. What is the domain of $y = e^x$?
   f. What is the range of $y = e^x$?
   g. What is the y-intercept of $y = e^x$?
   h. What is the x-intercept of $y = e^x$?
   i. Solve $e^x = 0$
   j. $\ln 1 = ?$
   k. $\ln e = ?$
   l. $e^{\ln x} = ?$
   m. $\ln e^x = ?$
   n. $\ln 0 = ?$
   o. Rewrite $2\ln x$ using properties
   p. Explain how $e^{x+\ln 3} = 3e^x$

4. What does it mean for a function to be odd? even? Answer these questions not only with what it means for the function but what it means with regards to its symmetry.

5. Which trig functions are even and which are odd? Which basic polynomials are even and which are odd?

6. Knowing which trig functions are even and odd should aid you in rewriting the following expressions in terms of x:
   a. cos(-x) = b. sin(-x) =

7. TRIG!!! You're going to love this part. You need to have each of the following values at your fingertips. There will be a timed quiz as part of the beginning of the year test on these values. You will also have surprise quizzes throughout the year.

a. sin 0	f. cos 0	k. tan 0
b. sin $\frac{\pi}{6}$	g. cos $\frac{\pi}{6}$	l. tan $\frac{\pi}{6}$
c. sin $\frac{\pi}{4}$	h. cos $\frac{\pi}{4}$	m. tan $\frac{\pi}{4}$
d. sin $\frac{\pi}{3}$	i. cos $\frac{\pi}{3}$	n. tan $\frac{\pi}{3}$
e. sin $\frac{\pi}{2}$	j. cos $\frac{\pi}{2}$	o. tan $\frac{\pi}{2}$

8. You should also be able to do trig stuff like this (this means you need to have the identities memorized! You won't get reference tables.):
   20 sin5x cos5x is equivalent to: *(this is a multiple choice question)*

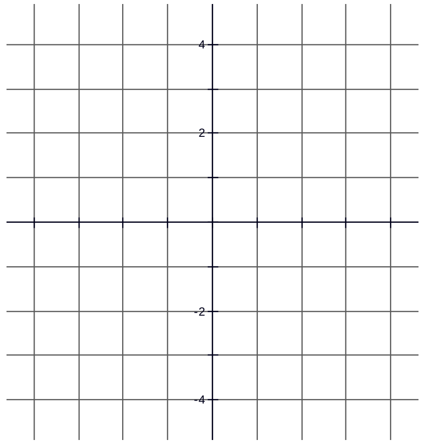
   a. 10sin10x
   b. 20sin5x
   c. 10sin5x
   d. 4cos5x
   e. 20cos5x

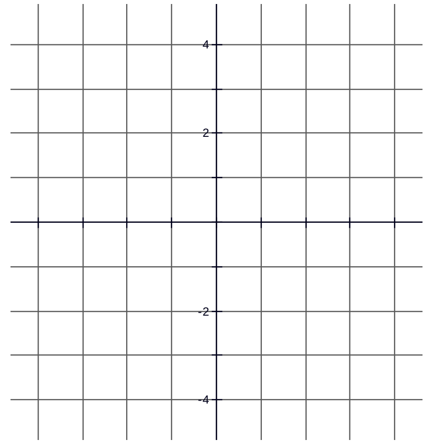
9. What do you know about a function and its inverse? (2 related things)

10. You should be able to quickly graph each of these basic functions without the aid of a calculator:

a. $y = x^2$

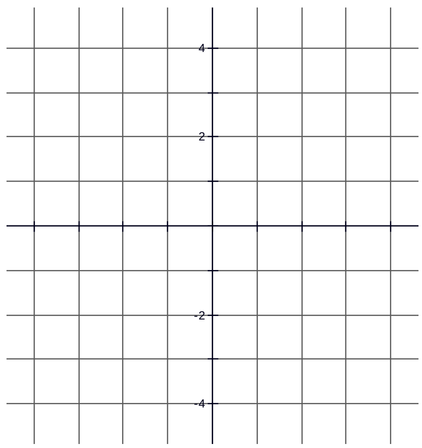
b. $y = x^3$

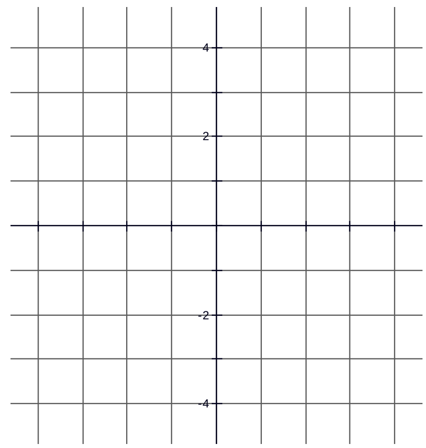




c. $y = \sqrt{x}$

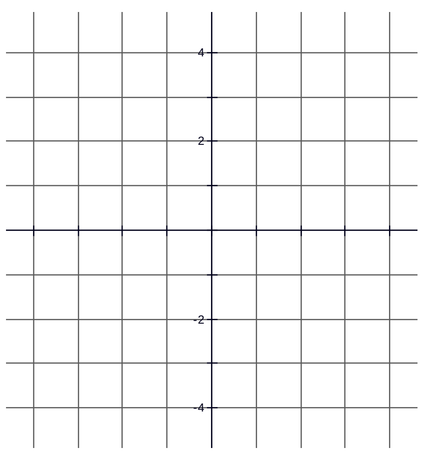
d. $y = |x|$

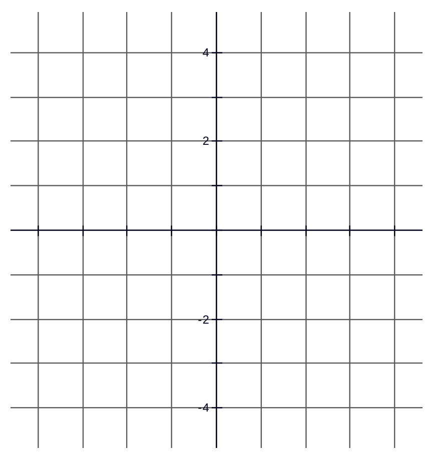




e. y = sin x

f. y = cos x

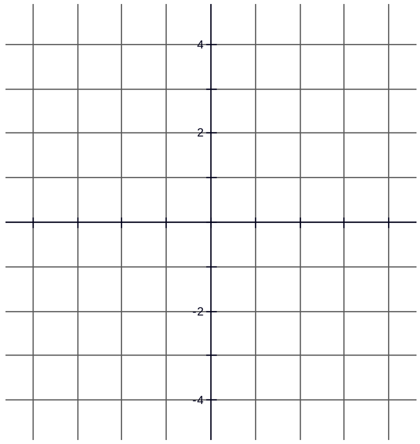


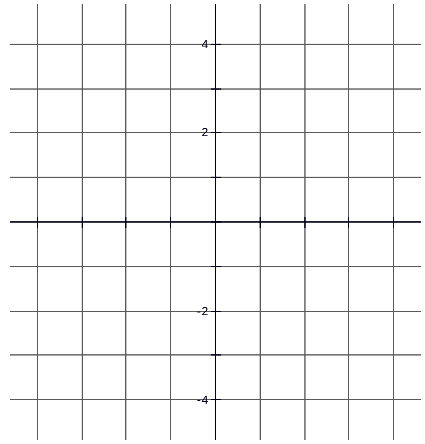


11. Graph each of these equations using your knowledge of the graphs of basic functions and their transformations without the aid of a calculator (think a, h, k!)

a. $y = 2x^2 - 4$

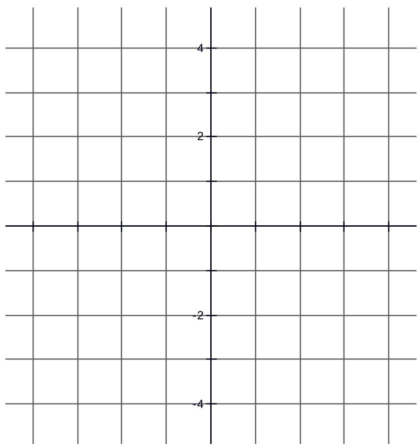
b. $y = 2(x-4)^2$

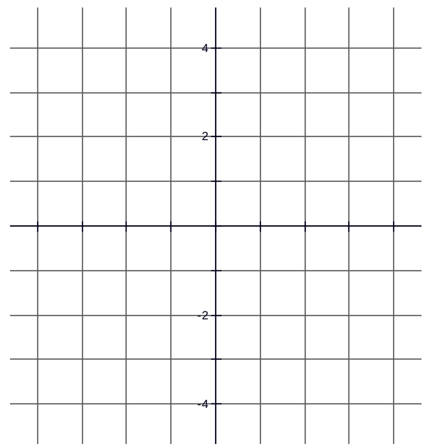




c. $y = \sin(2x) + 1$

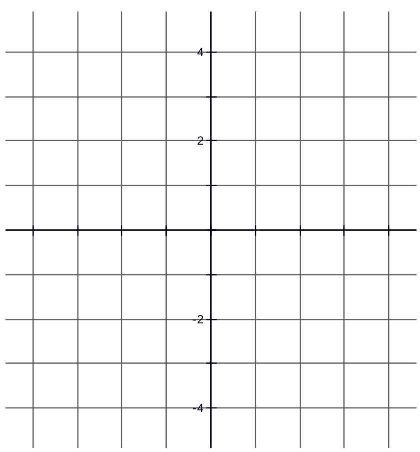
d. $y = (x+1)^3 - 3$

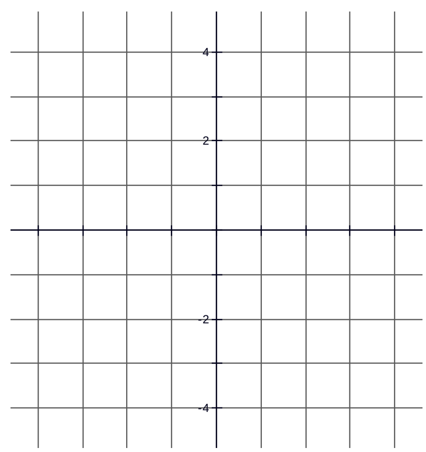




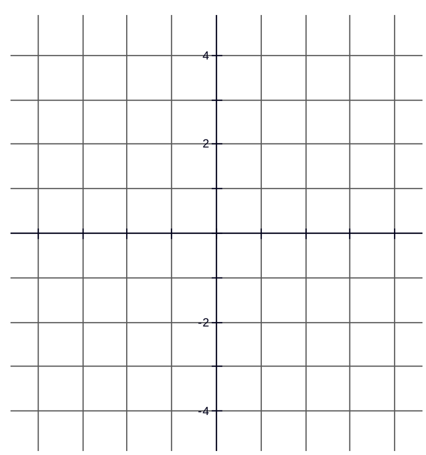
e. $y = \sqrt{x-5} + 3$

f. $y = 3|x+1|$





g. $y = 2\cos\left(x - \frac{\pi}{4}\right) - 5$

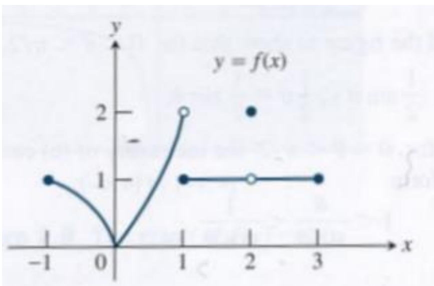


12. It is important too that you are able to understand and be able to write absolute value functions as a piecewise function.

a. $f(x) = |x|$

b. $f(x) = |x+3|$

13. For the function y = f(x) whose graph is shown below, determine each of the following limits.



$a. \lim_{x \to -1^+} f(x) =$ $b. \lim_{x \to 2} f(x) =$

$c. \lim_{x \to 2^+} f(x) =$ $d. \lim_{x \to 1^-} f(x) =$

$e. \lim_{x \to 1} f(x) =$ $f. \lim_{x \to 0-} f(x) =$

limits from a graph *Additional Problems: Ch 2 Review # 15-24*

14. Evaluate the following limits:

a. $\lim_{x \to -3} \frac{x}{x+3}$

b. $\lim_{x \to -\infty} \frac{2x^2+3}{5x^2+7}$

c. $\lim_{x \to -\infty} \frac{2x-1}{|x|-3}$

d. $\lim_{x \to \infty} \frac{2x-1}{|x|-3}$

e. $\lim_{x \to c} f(x) = \frac{3}{2}$

$\lim_{x \to c} g(x) = \frac{1}{2}$

(a) $\lim_{x \to c} [4f(x)]$

(b) $\lim_{x \to c} [f(x) + g(x)]$

(c) $\lim_{x \to c} [f(x)g(x)]$

(d) $\lim_{x \to c} \frac{f(x)}{g(x)}$

f. $\lim_{x \to 0} \frac{3\sin 4x}{x}$

Limits *Additional Practice: Ch 2 Review #1-14*

Part 2:

15. a. Using limits justify that y = $\frac{1}{x-3}$ has an infinite discontinuity at x = 3

b. Using limits justify that y = $\frac{x^2-9}{x-3}$ has a removable discontinuity at x = 3

c. Using limits justify that y = $\begin{cases} x & x<3 \\ x^2 & x\geq 3 \end{cases}$ has a jump discontinuity at x = 3

Pts of Discontinuity *Additional Practice Ch 2 Review # 25, 26, 29-32*

16. Prove the following limit using the epsilon-delta definition of a limit.

$$\lim_{x\to 3} x^2 = 9$$

Epsilon-Delta Definition of a Limit *Additional Practice Redo problems from worksheets in class*

17. What are the 4 types of non-differentiabilities and how do you justify them?

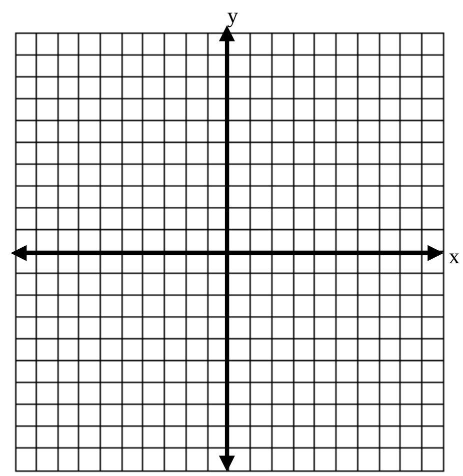
Differentiability *Additional Practice: Ch 3 Review #53, 55-58*

18. a. Find and justify the extreme values of the function on the interval and where they exist.

$g(x) = x^3 - 2x^2 + x - 1, \; -1 \le x \le 2$

b. For $y = x^4 - 72x^2 - 17$, use analytic methods to find:

a. intervals on which it is increasing
b. intervals on which it is decreasing
c. intervals on which it is concave up
d. intervals on which it is concave down
e. extreme values
f. inflection points
g. the graph of the equation



19. a. Given $f(x) = 5x^3 - 3x^2$,

i. Find the derivative of f(x) using the 2 limit definitions of a derivative.
ii. Find the equation of the tangent to the curve at x = -1.
iii. Find the equation of the normal to the curve at x = -1.

b. Write the equation of the tangent and normal line to the curve $2xy = x^2 + y^2$ at x = 3.

Defn Deriv *Additional Practice: Ch 3.1 #2,4*

Tangent and Normal Lines *Additional Practice: Ch 2 Rvw #47, 48 & Ch 3 Rvw #45-48,*

20. Find dy/dx and d^2y/dx^2:

a. $y = \frac{x^3}{3} + \frac{x^2}{2} + x$

b. $y = \frac{2x+1}{2x-1}$

c. $y^4 - y^2 = x^4$

21. Find dy/dx:

a. $y = \sqrt{x} + 1 - \frac{1}{\sqrt[3]{x}}$

b. $y = \ln\sqrt{x}$

(there's an easier way of doing this rather than using the chain rule, can you figure out how?)

c. $y = \sin^3 x \tan 4x$

d. $y = x^{\csc x}$

e. $y = e^x \sqrt{\tan 2x}$

f. $y = \cos^{-1} x^2$

g. $y = \log_3 (1 + x \ln 3)$

k. $x \sin 2y = y \cos 2x$

l. $x = y \tan^{-1} y$

22. The base of a pyramid-shaped tank is a square with sides of length 12 feet, and the vertex of the pyramid is 10 feet above the base. The tank is filled to a depth of 4 feet, and water is flowing out of the tank at the rate of 2 cubic feet per minute. How fast is the depth of water in the tank falling?

Related Rates *Additional Practice: Ch 4 Review #58-63*

23. Find the dimensions of the largest rectangle that can be inscribed in a semicircle of radius 5.

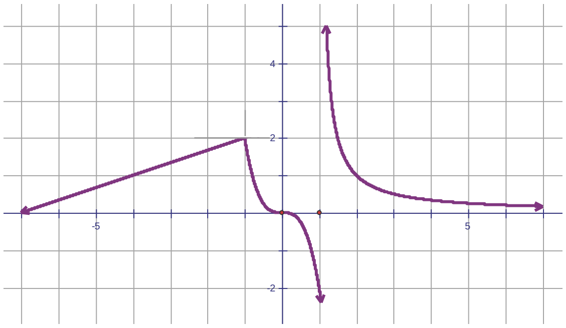
Optimization *Additional Practice: Ch 4 Review #46-57*

24. Show that $f(x) = x^2 + 2x - 1$ satisfies the hypotheses of the Mean Value Theorem on [0,1] and find the value of c that satisfies the hypothesis.

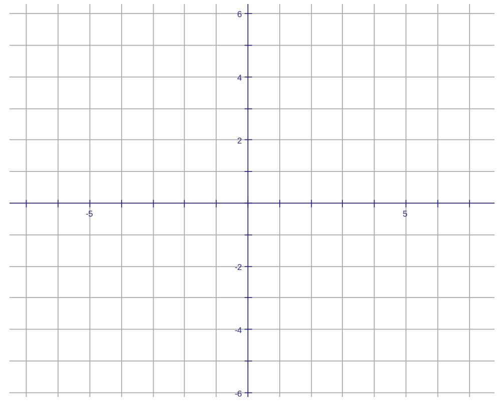
Mean Value Theorem *Additional Practice: Ch 4 Review #37*

25. a.Use the given graph of the derivative of the continuous function f(x) to sketch a graph of f(x) on x ϵ [-7,7] if f(1) = 0.

f '(x)

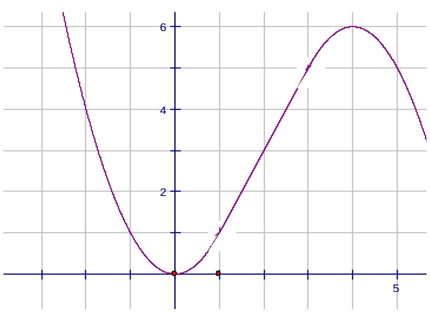


f(x)

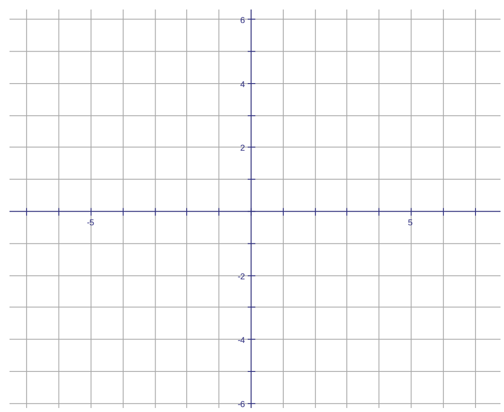


b. Use the graph of f(x) to sketch a graph of f '(x):

f(x)



f '(x)



Derivative Rules

f(x)	f′(x)	f(x)	f′(x)
x^n	nx^{n-1}	$\sin^{-1}x$	$\frac{1}{\sqrt{1-x^2}}$
sin x	cos x	$\cos^{-1}x$	$\frac{-1}{\sqrt{1-x^2}}$
cos x	- sin x	$\sec^{-1}x$	$\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$
tan x	$\sec^2 x$	$\csc^{-1}x$	$\frac{-1}{\lvert x\rvert\sqrt{x^2-1}}$
sec x	sec x tan x	$\tan^{-1}x$	$\frac{1}{1+x^2}$
cot x	$-\csc^2 x$	$\cot^{-1}x$	$\frac{-1}{1+x^2}$
csc x	- csc x cot x		
e^x	e^x		
a^x	$a^x \ln a$		
ln x	$\frac{1}{x}$		
$\log_b x$	$\frac{1}{x\ln b}$		

Product Rule

u·v → v·u′ + u·v′

-or-

f(x) · g(x) →
g(x) ·f′(x) + f(x) ·g′(x)

Quotient Rule

$$\frac{u}{v} \to \frac{v\cdot u' - u\cdot v'}{v^2}$$

-or-

$$\frac{f(x)}{g(x)} \to \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{(g(x))^2}$$

Chain Rule

f(g(x)) →
f′(g(x)) · g′(x)

-or-

f(u)→f′(u) · u′