AP Calculus
AB Summer Review

Name:_____

Date:_____

This packet is a list of things you should be able to do by the time you get into AP Calculus. These are all basic concepts that you should be able to do without a calculator coming into an AP Calculus class. If you are not fluent in them you will have difficulty learning the new material.

Click here for the key or use this: https://drive.google.com/open?id=0B8bL5nu2iZOlQTZDZ0Zidk40bE0

We will take a test on these concepts on the first Monday of the school year.

1. Use the binomial theorem or Pascal's triangle to expand each of the following polynomials:

   a. $(a + b)^4$

   b. $(x - 3)^3$

   c. $(1 - y)^6$

   d. $(2x + 1)^5$

2. a. Knowing the polynomial $2x^4 - 5x^3 - 6x^2 + 19x - 10$ has a double root at $x = 1$, use synthetic division to find the other roots.

   b. Use the rational zero theorem and synthetic division to find the roots of $x^3 + 3x^2 - 4x - 12$.

3. Find the value of each expression without the use of a calculator

a. $\sqrt[3]{8^4}$

b. $\sqrt{25^3}$

c. $\sqrt[5]{32^8}$

d. $\sqrt[3]{216^2}$

e. $\sqrt[4]{256^3}$

f. $\sqrt{49^3}$

4. Factor each expression completely

a. $x^3 + 8$

b. $x^3 - 27$

c. $8x^3 + 64$

d. $27x^3 - 125y^3$

5. Answer each of the following questions about logs and exponentials:

a. What is the domain of $y = \ln x$?
b. What is the range of $y = \ln x$?
c. What is the y-intercept of $y = \ln x$?
d. What is the x-intercept of $y = \ln x$?
e. What is the domain of $y = e^x$?
f. What is the range of $y = e^x$?
g. What is the y-intercept of $y = e^x$?
h. What is the x-intercept of $y = e^x$?
i. Solve $e^x = 0$
j. $\ln 1 = ?$
k. $\ln e = ?$
l. $e^{\ln x} = ?$
m. $\ln e^x = ?$
n. $\ln 0 = ?$
o. Rewrite $2\ln x$ using properties
p. Explain how $e^{x+\ln 3} = 3e^x$

6. What does it mean for a function to be odd? even? Answer these questions not only with what it means for the function but what it means with regards to its symmetry.

7. Which trig functions are even and which are odd? Which basic polynomials are even and which are odd?

8. Knowing which trig functions are even and odd should aid you in rewriting the following expressions in terms of x:

   a. cos(-x) = b. sin(-x) =

9. You should know each of the following. There will be a timed quiz as part of the beginning of the year test on these values. You will also have surprise quizzes throughout the year.

   Basic Unit Circle Ratios:

a.	$\sin 0$	f.	$\cos 0$	k.	$\tan 0$
b.	$\sin\frac{\pi}{6}$	g.	$\cos\frac{\pi}{6}$	l.	$\tan\frac{\pi}{6}$
c.	$\sin\frac{\pi}{4}$	h.	$\cos\frac{\pi}{4}$	m.	$\tan\frac{\pi}{4}$
d.	$\sin\frac{\pi}{3}$	i.	$\cos\frac{\pi}{3}$	n.	$\tan\frac{\pi}{3}$
e.	$\sin\frac{\pi}{2}$	j.	$\cos\frac{\pi}{2}$	o.	$\tan\frac{\pi}{2}$

   Reciprocal Function Ratios:

a.	$\sec\frac{\pi}{3}$	d.	$\cot\frac{\pi}{2}$	g.	$\sec^2\frac{\pi}{4}$
b.	$\csc\frac{-\pi}{3}$	e.	$\csc^2\frac{-\pi}{6}$	h.	$\cot\frac{\pi}{3}$
c.	$\sec\frac{2\pi}{3}$	f.	$\cot 0$	i.	$\csc\frac{\pi}{4}$

10. You should have a good understanding of inverse trig functions.
    a. What is the range of $\sin^{-1}x$?
    b. What is the range of $\cos^{-1}x$?
    c. What is the range of $\tan^{-1}x$?
    d. When you evaluate $\sin^{-1}x$, what quadrants does it give you the answer in?
    e. When you evaluate $\cos^{-1}x$, what quadrants does it give you the answer in?
    f. When you evaluate $\tan^{-1}x$, what quadrants does it give you the answer in?
    g. Why is sine positive in quadrants I and II but negative in III and IV?
    h. Why is cosine positive in quadrants I and IV but negative in II and III?
    i. Why is tangent positive in quadrants I and III but negative in II and IV?

11. You should be able to do problems like this as well:
    Solve the trig equation in the given interval:

    a. $\tan x = 1$ $\quad 0 \le x \le 2\pi$

    b. $\sec x = -2$ $\quad -\infty < x < \infty$

    c. $\sin x = -0.5$ $\quad -\pi \le x \le \pi$

12. You should also have the trig identities from precalculus committed to memory so you can answer questions like this-

    20 sin5x cos5x is equivalent to: *(this is a multiple choice question)*

    a. 10sin10x
    b. 20sin5x
    c. 10sin5x
    d. 4cos5x
    e. 20cos5x

13. You should also be able to simplify complex fractions:

    a. $\dfrac{\sqrt{2x-5}+\dfrac{6}{\sqrt{2x-5}}}{2x-5}$

    b. $\dfrac{1+x^2-\dfrac{x+3}{3}}{\dfrac{3x^3+2}{3}-1}$

    c. $\dfrac{\dfrac{x}{5}-\dfrac{5}{x}}{\dfrac{x}{5}-1}$

    d. $\dfrac{x^2+\dfrac{8}{x}}{x-\dfrac{4}{x}}$

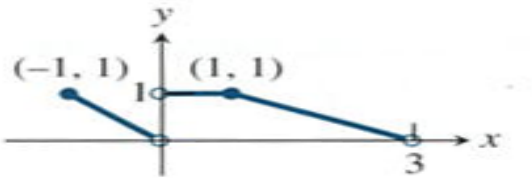
14. Describe two properties regarding the relationship between a function and its inverse?

15. You should be able to graph each of these basic functions without the aid of a calculator:
    a. $y = x^2$
    b. $y = x^3$
    c. $y = \sqrt{x}$
    d. $y = |x|$
    e. $y = \sin x$
    f. $y = \cos x$

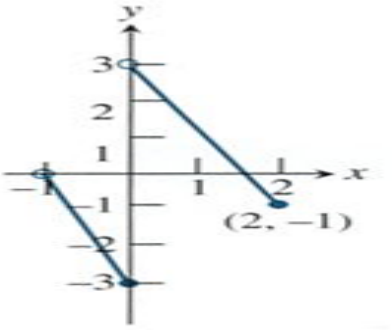
16. Graph each of these equations using your knowledge of the graphs of basic functions and their transformations without the aid of a calculator (think a, h, k!)
    a. $y = 2x^2 - 4$
    b. $y = 2(x-4)^2$
    c. $y = \sin(2x) + 1$
    d. $y = (x+1)^3 - 3$
    e. $y = \sqrt{x-5} + 3$
    f. $y = 3|x+1|$
    g. $y = 2\sin\left(x - \frac{\pi}{4}\right) - 5$

17. You also need to be able to write a piecewise function. Be able to write a piecewise function for each of the following:

a.



b.



18. You should also be able to graph a piecewise function. Graph each of the following:

a. $$f(x) = \begin{cases} 1 & x < 0 \\ \sqrt{x} & x \geq 0 \end{cases}$$

b. $$f(x) = \begin{cases} x^2 & x < 0 \\ x^3 & 0 \leq x \leq 1 \\ 2x - 1 & x > 1 \end{cases}$$

19. It is important too that you are able to understand and be able to write absolute value functions as a piecewise function.

a. $f(x) = |x|$

b. $f(x) = |x + 3|$