AP Calculus BC
Summer Assignment 2017

This assignment is meant to serve as a review of the topics, concepts, and skills necessary for the course next school year. The course is intense with an AP exam in early May. The list of topics in the curriculum is numerous and the level of difficulty is high. Therefore, class time is not devoted to reviewing prerequisite material (as well as going over any homework assignment during the year). Try not to save this assignment for Labor Day weekend. Budgeting your time is an important skill in life, let alone this course.

All work should be labeled and completed on separate paper (use graph paper for graphing) and should be submitted on the first day of class and will be factored into your grade. There will be an exam on Parts I, III, and IV within the first week of class. And a second exam on Part II and additional material about limits within the second week of class.

Part I: Review of Functions

For the following problems in parts A through H, state the domain and range for the following functions and sketch a graph (label axes). Do not plot points and do not use a graphing calculator (use only to check work). Instead analyze each function, using the algebraic definition of each function, transformations, the concept of limits, etc. Calculate x and y intercepts and asymptotes, holes (if any) and indicate on the graph.

A. Linear, Quadratic, Polynomial

Important concepts/ skills: Point slope form, factoring, completing the square, quadratic formula, synthetic substitution, properties of polynomial graphs (end behavior, type of roots)

1) $f(x) = x^2 + 2$
2) $f(x) = x^2 + 2x - 2$
3) $g(x) = x^3 - 4x^2 - 5x$

B. Absolute Value and piece-wise

Important concepts / skills: absolute value as a piece-wise function and using parent graphs / transformations, holes and break/ jump discontinuities

1) $f(x) = |3x - 2|$
2) $f(x) = -|3 - x| + 2$
3) $g(x) = \begin{cases} x-4, & x \le 5 \\ -x+2, & x > 5 \end{cases}$
4) $f(x) = \begin{cases} 3-x, & x \le 1 \\ 2x^2+1, & x > 1 \end{cases}$

C. Rational

Important concepts/ skills: Asymptotes (calculate using limits and via “short cuts”), holes, zero in numerator, zero in denominator (undefined, dne, or ∞), zero in both numerator and denominator

1) $f(x)=\dfrac{1}{x-2}$

2) $f(x)=1+\dfrac{1}{x}$

3) $f(x)=\dfrac{1}{x^2+2}$

4) $g(x)=\dfrac{x^2-x-6}{x^2-4x+3}$

5) Find the domain of $g(x)=\dfrac{4-x^2}{x^2+x}$ and solve $g(x)=0$.

D. Radical

Important concepts/ skills: The domain is all real numbers (we don’t work with imaginary numbers), parent graphs and transformations.

1) $f(x)=x^{\frac{1}{3}}$

2) $f(x)=x^{\frac{2}{3}}$

3) $f(x)=\sqrt[4]{-x}$

4) $f(x)=2+\sqrt{x-1}$

5) If $f(t)=\sqrt{t^2-16}$, find all values for t which $f(t)$ is a real number. Find $f(t)=3$

E. Exponential

Important concepts/ skills: properties of exponents, solving exponential equations, inverse of logarithmic function, parent graphs and transformations.

1) $f(x)=3^{-x}$

2) $f(x)=2e^{-x}-3$

3) $f(x)=-e^{x}+1$

F) Logarithmic

Important concepts/ skills: properties of logarithms, solving logarithmic equations, inverse of an exponential function, parent graphs and transformations.

1) $f(x) = \ln(x+1)$
2) $f(x) = \ln\left(x-3\right)+1$
3) $f(x) = \ln\left|x\right|$

G) Trigonometric

Important concepts/ skills: **Radians only** (we don't work with degrees), ASTC, evaluating any trigonometric expression using unit circle and/or special right triangles (i.e. an angle that is any multiple of $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$), solving equations, Pythagorean Identities (need to know and use), other identities: double and half angle formulas (not as important)

1) $f(x) = 2\sin(\pi x) + 1$
2) $f(x) = \tan(2x - \pi)$
3) $f(x) = \sec(x)$
4) State the domain and range and graph all 6 inverse trigonometric functions

H) Polar

Important concepts/ skills: convert from polar to rectangular coordinates, graphing on polar plane, knowing general equations for a circle, spiral, cardioid, limacon, and rose, and finding intersection points of 2 curves.

1) $r = 2 - 2\cos(\theta)$
2) $r = 3\sin(2\theta)$
3) $r = -6\cos(\theta)$
4) Find the intersection points of (a) and (c) on the interval $0 \le \theta \le 2\pi$

Part II. Limits

(Make sure to show all work for Part B)

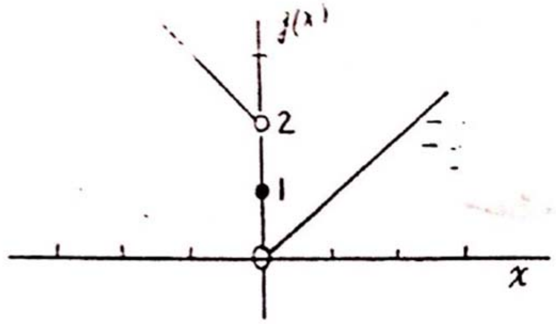
Part A: Limits Graphically

1. For the function g graphed at the right, find:

   a. $\lim_{x\to 0^-} g(x)$

   b. $\lim_{x\to 0^+} g(x)$

   c. $\lim_{x\to 0} g(x)$

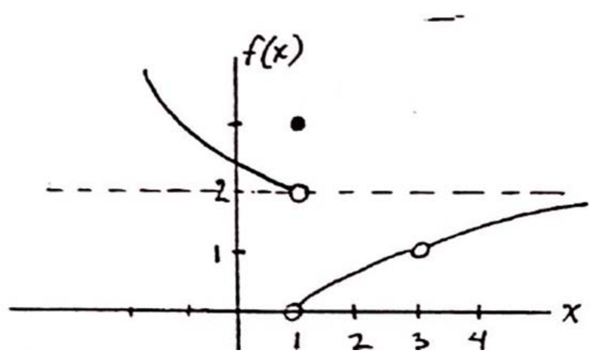


2. For the function f graphed at the right, find:

   a. $\lim_{x\to 1^-} f(x)$ d. f(1)

   b. $\lim_{x\to 1^+} f(x)$ e. $\lim_{x\to 3} f(x)$

   c. $\lim_{x\to 1} f(x)$ f. $\lim_{x\to +\infty} f(x)$



3. For the function g graphed at the right, find:

   a. $\lim_{x\to +\infty} g(x)$ f. $\lim_{x\to 4^-} g(x)$

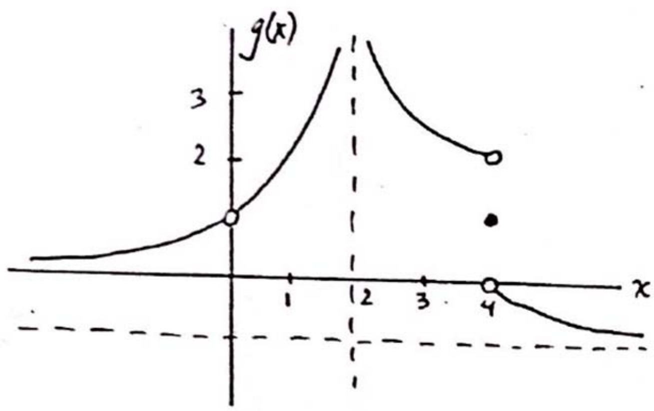
   b. $\lim_{x\to -\infty} g(x)$ g. $\lim_{x\to 4^+} g(x)$

   c. $\lim_{x\to 0^-} g(x)$ h. $\lim_{x\to 4} g(x)$

   d. $\lim_{x\to 0^+} g(x)$ i. g(4)

   e. $\lim_{x\to 0} g(x)$ j. $\lim_{x\to 2} g(x)$

   g(0) und.



4. For the function f graphed at the right, find:

a. $\lim_{x \to 2^-} f(x)$

b. $\lim_{x \to 2^+} f(x)$

c. $\lim_{x \to 2} f(x)$

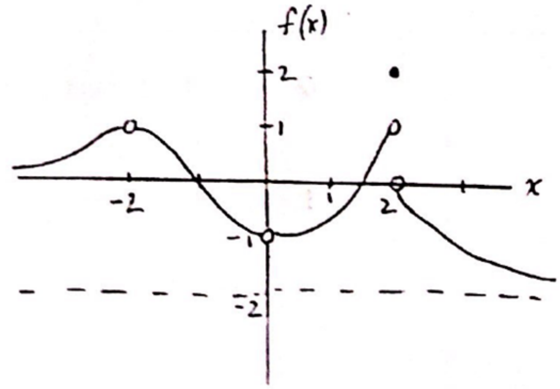
d. f(2)

e. $\lim_{x \to 0} f(x)$

f. $\lim_{x \to -2} f(x)$

g. $\lim_{x \to -\infty} f(x)$

h. $\lim_{x \to +\infty} f(x)$



by B. Zentele

Part B: Evaluating limits

Evaluate each limit:

1. $\lim\limits_{x \to 2} \dfrac{x-1}{x^2-1}$

2. $\lim\limits_{x \to 1} \dfrac{x-1}{x^2-1}$

3. $\lim\limits_{x \to 1} \dfrac{x^2-1}{x+1}$

4. $\lim\limits_{x \to 1} \dfrac{x+1}{x^2-1}$

5. $\lim\limits_{x \to 1} \dfrac{x^2-6x+5}{x^2+x-2}$

6. $\lim\limits_{x \to 0} \dfrac{5x^3+8x^2}{3x^4-16x^2}$

7. $\lim\limits_{x \to 1} \dfrac{x-1}{\sqrt{x^2+3}-2}$

8. $\lim\limits_{x \to 0} \dfrac{1}{1-\frac{1}{x}}$

9. $\lim\limits_{x \to 4} \dfrac{\frac{1}{\sqrt{x}}-\frac{1}{2}}{x-4}$

10. $\lim\limits_{x \to a} \dfrac{\sqrt{x}-\sqrt{a}}{x-a}$

11. $\lim\limits_{x \to +\infty} \dfrac{2x+3}{5x+7}$

12. $\lim\limits_{x \to +\infty} \dfrac{3x^2-6x}{2x-5}$

13. $\lim\limits_{x \to -\infty} \dfrac{7x-12}{x^2}$

14. $\lim\limits_{x \to +\infty} \dfrac{\sqrt{x+1}}{\sqrt{4x-1}}$

15. $\lim\limits_{x \to +\infty} \dfrac{4+x}{\sqrt{4x^2+1}}$

16. $\lim\limits_{x \to -\infty} \dfrac{4+x}{\sqrt{4x^2+1}}$

17. $\lim\limits_{x \to +\infty} \dfrac{\sqrt{4x^2+1}}{3x-6}$

18. $\lim\limits_{x \to 0^+} \dfrac{|x|}{x}$

19. $\lim\limits_{x \to 2^-} \dfrac{|x-2|}{x-2}$

20. $\lim\limits_{x \to -1} \dfrac{|x|-1}{x+1}$

21. $\lim\limits_{x \to 3^+} \dfrac{1}{x-3}$

22. $\lim\limits_{x \to 3} \dfrac{1}{x-3}$

23. $\lim\limits_{x \to 1} \dfrac{x}{(x-1)^2}$

24. $\lim\limits_{x \to 1^+} \sqrt{x-1}$

25. $\lim\limits_{x \to 1^-} \sqrt{x-1}$

26. $\lim\limits_{x \to 1^+} 2^{\frac{1}{x-1}}$

27. $\lim\limits_{x \to 1^-} 2^{\frac{1}{x-1}}$

28. $\lim\limits_{h \to 0} \dfrac{(x+h)^2-x^2}{h}$

29. Given $f(x) = x^2 + x$

a) $\lim\limits_{x \to 1} \dfrac{f(x)-f(1)}{x-1}$

b) $\lim\limits_{h \to 0} \dfrac{f(1+h)-f(1)}{h}$

30 For what value of k does $\lim_{x \to 2} \frac{x^2 - kx + 6}{x - 2} = -1$

31. Given $f(x) = \begin{cases} x, & x \le 0 \\ x + 1, & x > 0 \end{cases}$. Explain why $\lim_{x \to 0} f(x)$ does not exist.

Part III: Review – Definition of the Derivatives

A. Solve the following using the definition "the limit as x approaches a".

1. Find $f'(1)$ of $f(x) = 3x^2 + 5x$. Describe what this means graphically.

2. Find $\left.\frac{dy}{dx}\right|_{x=20}$ of $f(x) = x$. Describe what this means as a "rate of change".

3. Find the slope of the tangent line to the curve $f(x) = \frac{1}{x^2}$ at $(1,1)$.

4. Write an equation of the line tangent to $y = \sqrt{x+1}$ at the point x = 8.

5. Find $f'(2)$ of $f(x) = x^3 - 4x^2 - 9x + 46$.

B. Solve the following using the definition "the limit as h approaches zero".

1. Find $f'(-1)$ of $f(x) = 3 - x^2$. Describe what this means graphically.
2. Find $\left.\frac{dy}{dx}\right|_{x=8}$ of $f(x) = -2x + 3$. Describe what this means as a "rate of change".
3. Find the slope of the tangent line to the curve $f(x) = -\frac{1}{x}$ at $(\frac{1}{2},\text{-}2)$.
4. Write an equation of the line tangent to $y = \sqrt{1-x}$ at the point $x = -3$.
5. Write the equation of the line tangent to the function $f(x) = x^3$ at the point $x = -2$

C. Estimating values for the derivative of a function at a point.

1. Estimate the derivative (i.e. find $\frac{\Delta y}{\Delta x}$) of $f(x) = e^x$ at $x = 2$ using a table of values (to three decimal places).

2. The following table shows values for $f(x) = x^3$ near $x = 2$. Use it to estimate $f'(2)$.

x	1.998	1.999	2.000	2.001	2.002
$f(x)$	7.976	7.988	8.000	8.012	8.024

Part IV: Rectilinear Motion

The derivative calculates an "instantaneous rate of change". A popular application of" rates of change" in this course is in the area of motion of an object (Physics).

*For students who did not take Physics this material will be taught / reviewed during class.

Important concepts/ vocabulary: scalar/ vector quantities, position, velocity, speed, acceleration (positive vs negative), speeding up/ slowing down (decelerating), displacement/ distance, average vs instantaneous

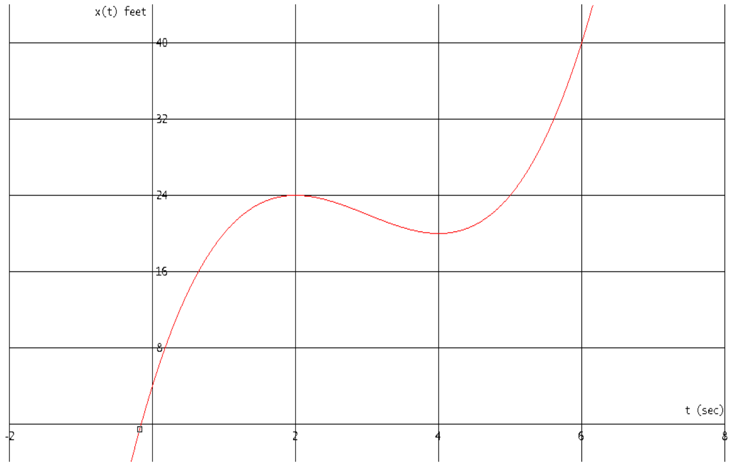
Part I

1) Draw a sketch of a position-time curve for a particle moving along a coordinate axis with the following characteristics.
   a) Particle is on the positive side of the origin.
   b) Particle is moving in a negative direction.
   c) Particle velocity is decreasing.
   d) Particle is speeding up.

2) Draw a sketch of a position-time curve for a particle moving along a coordinate axis with the following characteristics.
   a) Particle is on the negative side of the origin.
   b) Particle is moving in a positive direction.
   c) Particle velocity is increasing.
   d) Particle is speeding up.

3) Draw a sketch of a velocity-time curve for a particle moving along a coordinate axis with the following characteristics.
   a) Particle has a negative velocity.
   b) Particle velocity is increasing.
   c) Particle is slowing down.

4) Suppose a particle moves on the x-axis so that its position (x) at time (t) is given by $x(t) = t^3 - 9t^2 + 24t + 4$. Time is measured in seconds and distance is measured in feet.



a) When is the particle moving forward?

b) When is the particle at rest?

c) Starting at $t = 0$, how far right does the particle reach before it reverses its motion?

d) What is the total displacement in the first 6 seconds?

e) What is the total distance traveled in the first 6 seconds?

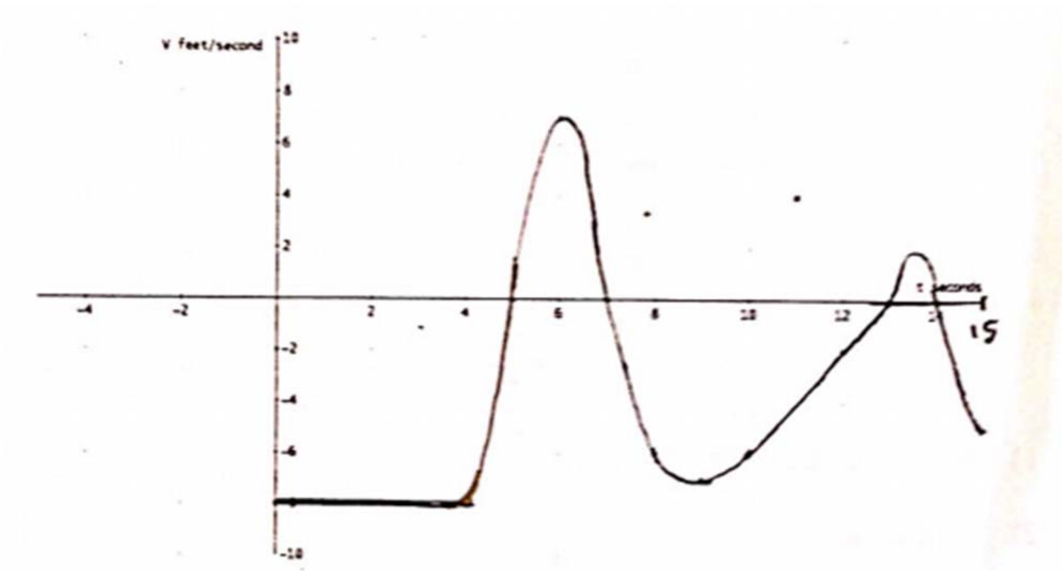
f) During what time period(s) is the particle's velocity positive? When is it negative?

g) During what time period(s) is the particle's velocity increasing (accelerating)? When is it decreasing (negative accelerating)?

h) When is the particle speeding up? When is it slowing down (decelerating)?

i) Draw a sketch of a velocity time graph for the particle.

5) Derek Jeter is doing some agility exercises during spring training. He runs back and forth between third base and home. The team trainer is standing at home plate with a motion detector pointed at Jeter. He takes the data from his motion detector and plots a graph of Jeter's velocity for a 15 second time interval. The graph of the data is below. A positive velocity indicates Jeter's movement away from home and a negative velocity shows his movement toward home.



When is Jeter...

a) heading towards home?

b) returning to third base?

c) changing directions?

d) accelerating positively?

e) accelerating negatively?

f) speeding up?

g) slowing down (decelerating)?

h) Jeter's velocity at $t = 8$ seconds?

i) Jeter's speed at $t = 8$ seconds?

j) **Estimate** Jeter's acceleration at $t = 6$ seconds?

k) **Estimate** Jeter's acceleration at $t = 13$ seconds?