# CCSS *Standards for Mathematical Practice* in CPM *Core Connections* Courses

Although CPM predates the CCSS *Standards for Mathematical Practice* by about 20 years, very similar mathematical practices have always been core and integral parts of CPM curriculum. Ever since its inception in 1989, the CPM curriculum has taken its principles of course design from methodological research in teaching mathematics: the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections; and more recently the National Research Council *Adding It Up* findings for proficiencies in adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition.

Because of CPM's broad experience with similar mathematical practices, the CCSS *Standards for Mathematical Practice* are deeply woven into the daily lessons. For details regarding the principles of course design used in developing the CPM *Core Connections* courses, see the "Program Description."

## Mathematical Practices are a Core and Integral Part of CPM

**1. Make sense of problems and persevere in solving them.**
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

The *Core Connections* courses have students solve realistic, non-routine problems that are rich in mathematics on a daily basis. These guided investigations are not mere "word problems" that mimic algorithmic, computational problems. By having students make sense of the problem, rather than being told how to solve a particular kind of problem step by step, CPM *Core Connections* helps students to develop deep conceptual understanding of the mathematics, procedural fluency, and perseverance. In addition, the curriculum teaches problem-solving strategies that the students continue to use. The curriculum equips students with strategic competence and adaptive reasoning.

 

**2. Reason abstractly and quantitatively.**
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Instead of offering word problems only at the end of each chapter or problem set, the CPM curriculum generally presents mathematical ideas in contexts *first*, helping students make sense of otherwise abstract principles. Only then do students move on to abstraction and generalization using symbolic notation. Students are taught to try specific numbers to evaluate what is happening in problems, critically consider units and analyze what a number really means in a problem, find a number sentence that represents a given relationship, and identify patterns and relationships that lead to solutions. Students are also asked to work in reverse, that is, to create situations for abstract generalizations.

**3. Construct viable arguments and critique the reasoning of others.**
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

In CPM *Core Connections* courses, students regularly share information, opinions, and their knowledge and understandings in study teams. They take turns contributing, listening, arguing, asking for help, checking for understanding, and keeping each other focused. During this process students use higher-order thinking: providing clarification, building on each other's ideas, analyzing and coming to consensus, and productively criticizing. Justifying and critiquing happens every day in a CPM classroom; it does not occur as an occasional assignment. Students must communicate their mathematical findings in a clear and convincing manner, via narratives, oral presentations, or posters. Teachers answer students' questions, doing so in a way that challenges and motivates students to conjecture and test solutions themselves.

 

**4. Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Modeling contextual situations with multiple representations recurs throughout the CPM *Core Connections* series. From their first encounters with basic functions like proportions and linear equations, students learn to model using tables, graphs, equations, written descriptions, and diagrams. They continue to practice continues throughout each course, but with increasingly complex functions. In doing so, students make assumptions, form predictions, and then check to see if their predictions make sense in the context of the problem. Students use area models to multiply fractions, multiply and divide polynomials, factor, and solve probability problems. When challenged with variability in data, students learn that models may not be perfect, yet can be very useful for describing data and making predictions. CPM students find that using a calculator or computer can help them model repeated probabilistic experiments much more efficiently than actually conducting the experiment.

**5. Use appropriate tools strategically.**
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

In the typical CPM *Core Connections* lesson, students have several available tools—including rulers and scissors, tracing paper and graph paper, blocks and tiles, and eTools and calculators—but are not typically told which specific tools to use to solve any particular problem. Each teams of students usually has a designated Resource Manager, whose task it is to ask the teacher for the tools the team needs for that lesson. It is not unusual for different teams to use different tools to solve a problem. During a typical lesson, students share their solution strategies with the whole class, which frequently provokes a lively discussion of which tools were most efficient and productive to solve a given problem. For problems intended to help students become fluent with algebraic procedures, the CPM *Core Connections* texts use an icon to indicate calculators should not be used. But while conducting investigations students may choose to explore with their calculators to make sense of the mathematics without getting bogged down in computations. In some lessons, students explore concepts using appropriate technology tools, such as motion detectors, calculator programs, and electronic tools (for example HTML5, java, and Flash).

**6. Attend to precision.**
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Since they are solving contextual problems on a daily basis, CPM students understand the need for attending to precision. Students in this course must communicate clearly, giving carefully formulated explanations to each other. While working on problems, students pay attention to units, converting units to be consistent. They also check whether a numerical solution makes sense. Other times they are precise in scaling or reading the scale on a graph. The use of calculators in some CPM investigations requires students to attend to the precision of the results displayed.

 

### 7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

Since CPM students are developing conceptual understanding of the underlying mathematics, they frequently look for and make use of structure when bringing closure to investigations. For example, cross-multiplying to find equivalent fractions is not taught simply as a procedure to be practiced, but is developed from the underlying structure of a multiplication table. Students make connections between proportions, growth, steepness, and slope by exploring these as different structures of rates. CPM students do not simplify rational expressions by "canceling"; instead they use the underlying structure of the "Giant One"—fractions where the numerator and denominator are equal. Rather than listing geometric theorems in isolation these are developed using the structure of repeated translations. Polynomials are multiplied and divided using the structure of an area model. Moreover, polynomials equations are not solved by just following a set of steps, but by looking at the structure of the factored form and determining the type of root associated with that structure.

### 8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

When faced with a new investigation of a mathematical concept, CPM students often look for a simpler or analogous problem. By extending the structures used when solving previous problems, students are able to solve increasingly complex problems. CPM students use repeated reasoning in multiplication tables to multiply fractions or find equivalent fractions. Students expand the reasoning of simpler intuitive probability problems into increasingly more complex probabilistic situations. CPM students observe repeated structure in area models and using it to multiply, factor, and eventually divide, polynomials. Students use repeated patterns to make sense of negative, zero, and fractional exponents, and to solve rational expressions. Repeated reasoning allows for increasingly complex geometric proofs to be developed from simpler ones.

# Integration of the CCSS *Standards for Mathematical Practice* in a Sample Lesson

A typical CPM lesson reveals depth of integration of the CCSS *Standards for Mathematical Practice* into the course. In Lesson 9.1.1 of CPM's *Core Connections, Course 2*, students work in collaborative teams and as a class to discover the proportional relationship between the diameter and circumference of a circle. The lesson launches (problem 9-1, "Bubble Madness,") with the teacher asking students to predict how the length of a string wrapped around a can of tennis balls would compare to the height of the can. Students make predictions by imagining and visualizing (Practice 2), not by actually measuring, and then share predictions in a brief class discussion (Practice 3). Students then move into the main activity for the day, which will help them answer this question (Practice 1). In problem 9-2, the students work in teams to gather data (Practice 4) in order to determine the relationship between the circumference of a circle and its diameter. They do this by blowing bubbles with a soap solution and then letting the bubbles pop on a piece of construction paper, which leaves a clear circle. Students then measure the distance around and across the circle with string and a ruler or meter stick (Practices 5 and 6). Once they have gathered data for at least 8 different circles students organize their data in a table and create an appropriate graph (Practices 4 and 5). In order to make the graph students need to locate tenths of centimeters, which requires them to reason quantitatively (Practice 2). In problem 9-3, students turn to interpreting the pattern or relationship in the data on their graph (Practices 7 and 8). It is particularly important in this step for students to recall that their graph is based on measurements and will necessarily include a degree of error (Practice 6). The teacher leads a whole class discussion about whether the two measurements are related proportionally, drawing upon knowledge the students have developed in previous lessons regarding the characteristics of proportional relationships in tables and graphs (Practices 2 and 3). In the remainder of the lesson students connect their investigation discoveries to formal definitions and measurements of $\pi$ (Practice 6) and work to generalize the relationship between circumference and diameter (Practice 2). In problem 9-7, students return to the opening question, decide if their prediction has changed and discuss this with their team (Practice 3). Then they test their prediction (Practices 4 and 5). The final problem of the lesson, problem 9-8, contains applications and extensions of what students have learned about circumference and diameter (Practice 7). For the "Closure" activity of the lesson, the class discusses and then each student formally records the generalization of the relationship (Practices 3, 4, and 7).

 

# Correlation of CPM *Core Connections, Course 2*
# to CCSS Mathematical Practices

The *CCSS Standards for Mathematical Practice* are a core and integral part of all the lessons in CPM *Core Connections, Course 2*. The Practices are not an occasional activity in CPM, nor are they simply tacked on to the lessons.

The table below offers a list of sample lessons that the reader can review to see how the Mathematical Practices are embedded in all the lessons. The reader should also examine the detailed sample lesson on the previous page before examining any of these lessons. The table is by no means exhaustive; it only illustrates how the practice are integrated into a few typical lessons. The *CCSS Standards for Mathematical Practice* are deeply and seamlessly interwoven into the fabric of <u>each</u> of the daily lessons. An "xx" in the table represents a practice that is a focus of the lesson. An "x" represents a practice that is present in the lesson.

In addition to the daily execution of the Practices as exemplified in the table below, culminating problems at the end of the course ask students to specifically reflect on when, where, and how they used the Practices during the course (problems 9-104, 9-107, 9-111).

## Correlation of CPM *Core Connections, Course 2*
## to the CCSS *Standards for Mathematical Practice*

| | **Mathematical Practice** | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **CPM *Core Connections, Course 2* Lesson #.#.# Title** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** |
| 2.2.1 Composing Integers | x | xx | | x | x | | x | |
| 2.3.1 Choosing a Scale and Graphing Data | x | | x | x | x | x | x | |
| 3.1.1 Grouping Expressions | xx | xx | x | x | x | x | xx | x |
| 3.2.2 Addition and Subtraction | | x | x | x | x | | x | xx |
| 3.1.2 Scale Drawings | x | x | | x | | x | x | |
| 4.2.2 Proportional Relationships with Tables and Graphs | x | x | | x | x | | x | |
| 4.3.1 Combining Like Terms | | x | | xx | xx | | x | |
| 5.2.4, 5.2.5 Probability Tables and Trees | | x | | xx | x | | xx | |
| 5.3.2 Solving a Word Problem | x | | | | x | | x | x |
| 6.2.1 Solving Equations | x | x | x | x | x | | | |
| 6.2.4 Use a Table to Write Equations from Word Problems | x | | | x | x | | xx | |
| 7.1.2 Scaling Quantities | x | x | | xx | x | x | | |
| 7.2.1 Finding Missing Information in Proportional Relationships | x | | x | x | x | | | x |
| 8.1.1 Measurement Precision | x | x | x | x | x | xx | x | |
| 8.3.1 Introduction to Angles | x | x | | x | x | x | | |
| 9.1.1 Circumference, Diameter, Pi | x | x | x | x | x | x | x | xx |
| 9.2.2 Cross Sections | x | x | x | x | x | | | |

 